Recreational Mathematics Colloquium VI—G4G Europe

BOOK OF ABSTRACTS

National Museum of Natural History and Science, Lisbon
January 26th—January 29th, 2019

Organization:

Ludus Association
National Museum of Natural History and Science, Lisbon
Center for Mathematics and Fundamental Applications
Interuniversity Center for the History of Science and Technology
CEMAPRE—Centre for Applied Mathematics and Economics, ULisboa

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Richard Nowakowski (Dalhousie University, Canada)
Robin Wilson (Open University, England)
Thane Plambeck (Counterwave, inc, United States)

Foreword

Two years elapsed and time has come to gather and celebrate Recreational Mathematics again. In 2019, however, we got two ephemerides: Raymond Smullyan would turn 100 this year. We all miss him very much, no matter how many fantastic books and memories he left behind. We hope that, from wherever he is, Raymond will join us at the Colloquium. Our first RMC was held in Évora in 2009. We are turning 10! And, yes, getting younger in the process!

RMC-VI (G4G–Europe) will be another sophisticated and fun show-and-tell of mathematical beauties! We thank our visitors and hope our organizational skills will rise to the occasion.

Ludus Association, with the kind support of MUHNAC, ULisboa, CMAF-IO, CIUHCT, CEMAPRE, and FCT is proud to organize the sixth colloquium in the series, the third integrated in the Gathering for Gardner movement.

You can get more information at `http://ludicum.org/ev/rm/19`

The organization

Program

Saturday, 26th January

9:00 **Opening Session**

9:30 **The bridges of Koenigsberg**
David Singmaster, London South Bank University

10:00 **Raymond Smullyan's legacy: A centennial celebration**
Carlos Pereira dos Santos, CEAFEL, University of Lisbon

10:30 **The magic of the periodic table**
Fernando Blasco, Universidad Politécnica de Madrid

11:00 Coffee-Break

11:30 **A matter of perspective**
Richard Nowakowski, Dalhousie University

12:00 **Tiling boards**
José Carlos Santos, University of Oporto

12:30 **On the probability equilibrium of the Egyptian game Senet**
Luís Trabucho, FCT-UNL

13:00 Break for lunch

14:30 **Euler's formula and the Pascal triangle**
Pedro Freitas, FCUL, University of Lisbon

15:00 **Carl Friedrich Gauss and the formula for the sum of the first n natural numbers**
Aleksandra Ravas

15:30 **How a magician hacks the brain**
Adrien Lochon

16:00 Coffee-Break

16:30 **Pythagoras, Archimedes, and the napkin ring**
Colin Wright, Solipsys Ltd

17:00 **No gardeners like M. Gardner**
Guido Ramellini, MMACA

17:30 **A gambling portrait: cheating devices and their story**
Francisco Mousinho, ESTALs

18:00 **Magnums: Counting Sets with Surreals**
Peter Lynch, University College Dublin

19:00 **Conference dinner**

Sunday, 27th January

9:30 **Lewis Carroll's "A tangled tale"**
Robin Wilson, Open University, UK

10:00 **The sweet algorithm**
Hugo Almeida

10:30 **A reflection on the ideas of Miguel de Guzman about games and mathematics teaching**
Jaime Carvalho e Silva, University of Coimbra

11:00 Coffee-Break

11:30 **"Contas de cabeça" - a book of mathematical riddles edit by FPF**
Hélder Pinto, CIDMA-UA & Cristina Silva, ES Pinhal Novo

12:00 **An innovative card trick that combines classical concepts**
Ricardo Teixeira, University of Houston, Victoria

12:30 **"The book of numbers": Unveiling sequences**
Francisco Albuquerque Picado, ULisboa

13:00 Break for lunch

14:30 **The numerical calculation of God: a humorous text by Boris Vian**
Carlota Simões & Cristina Cordeiro, University of Coimbra

15:00 **The biggest secrets in the world**
Rogério Martins, FCT

16:00 **Open Session | The Maths and Magical Cabaret**
Chimico

During the course of an hour and a half we aim to entertain you with various acts taken from Magic, Maths and their intersection. All presented in the National Museum of Natural History and Science*. Hosted by the Circo Matemático we'll have various performers from various places of the globe, each with their unique act and skills.

Adrien Lochon | France
Colin Wright | Australia
Circo Matemático | Portugal
Francisco Mousinho | Portugal
Ricardo Teixeira | Brazil
Sydney Weaver |United States of America

*A entrada para o espectáculo é gratuita / Admission to the show is free.

Monday, 28th January

9:30 **Are the Chinese Rings Chinese?**
Andreas Hinz, LMU Munich

10:00 **NMATH**
Rodrigo Girão Serrão, MMA-IST

10:30 **Defending with one or three dice: What are the RISKs?**
Flavia Sancier-Barbosa, Colorado College

11:00 Coffee-Break

11:30 **The magical mystery Thue-Morse**
Jorge Buescu, ULisboa

12:00 **Topomagic, always the same trick?**
Tiago Hirth, Ludus

12:30 **Popularizing the first seventeenth century books of recreational mathematics: the "Ozanam Project"**
Lisa Rougetet, Université de Bretabne Occidentale

13:00 Break for lunch

14:30 **Cyclic surprises**
José Paulo Viana

15:00 **History of recreational mathematics**
Tereza Bártlová, Charles University

15:30 **Palindromes**
Joaquim Eurico Nogueira, FCT-UNL

16:00 Coffee-Break

16:30 **High dimensioned Gosper-like space filling curves**
Andrew Shirtz

17:00 **Mathematical Circus clown tricks**
Andreia Hall, Paulo Almeida, Sónia Pais & Paolo Vettori, University of Aveiro

17:30 **Cumulative Subtraction Games**
Urban Larsson, Technion, Israel Institute of Technology

Tuesday, 29th January

9:30 **Urban sketching with spherical perspective**
António Araújo, UAb

10:00 **Symmetry in the walls of the Palace of Ramalhão**
Alexandre Mena e Silva, CSJ

10:30 **Who wins the Gambling Tic-Tac-Toe?**
Ravi Kant Rai, IEOR, IIT Bombay

11:00 Coffee-Break

11:30 **Real Life Problems and a Vast Conspiracy**
Adam Atkinson

12:00 **A Mathematical Modeling Activity: optimization and planning of operating room**
Felipe Cabral, University of Coimbra

12:30 **Bemposta Palace, Military Academy Museums and laboratory**
Filipe Papança, Military Academy

13:00 Break for lunch

Auditório Caleidoscópio, Campo Grande

15:00 Public session **Our dearest problems**

Chair: Jorge Buescu (CMAF-IO)

Adam Atkinson: *The subheaps problem*

Andreas M. Hinz: *The Linear Tower of Hanoi*

Guido Ramellini: *Big box*

Ricardo Teixeira: *Collatz conjecture*

Thane Plambeck: *Guessing Red or Black*

10 minutes each. At the end of all presentations there will be a Q&A period.

16:30 Closing, Presence of Professor António Feijó, Vice-Rector of the ULisboa

Abstracts

The bridges of Koenigsberg
David Singmaster, London South Bank University

The Bridges of Konigsberg are one of the most famous topics in mathematics, but there is very little on the history of the actual bridges. I will show some pictures of actual bridges in the early 20C and show which bridges existed at various times. At most times an Eulerian Path existed and this remains the case.

Raymond Smullyan's Legacy: A Centennial Celebration
Carlos Pereira dos Santos, ISEL-IPL & CEAFEL-UL
Alda Carvalho, ISEL-IPL & CEMAPRE-UL
Jorge Nuno Silva, CIUHCT, ULisboa

Raymond Merrill Smullyan (1919–2017) was an American mathematician, magician, pianist, logician, Taoist, philosopher, and amateur astronomer. He was a brilliant logician, having significantly contributed for a better understanding of Gödel's theorem. Smullyan, a close friend of Martin Gardner, was one of the most relevant recreational mathematicians that ever lived, having published many masterpieces on recreational logic. Books like *What is the name of this book?*, *The chess mysteries of the Arabian knights*, and *The riddle of Scheherazade* are pearls in this field. The Island of Knights and Knaves and chessboards where detectives and kings solve problems using retrograde analysis are two magic realms of pure creativity. 2019 marks 100 years since the birth of Raymond Smullyan. In this talk, we will show some examples of his outstanding recreational work.

The magic of the periodic table
Fernando Blasco, Universidad Politécnica de Madrid

In 2018 the CL anniversary of the creation of the Periodic Table by Dimitri Mendeleiev will be celebrated. This lecture, prepared as a joint work with Miquel Duran, is thought as an approach to the work of this scientist by means of legerdemain magic tricks. We have adapted some mathematical magic tricks to periodic table tricks using the fact that in the PTE natural numbers appear in a straightforward way using atomic numbers. It will content card tricks (as those cards helped Mendeleiev to think on the way to order the chemical elements), decks with symbols of the elements and another tricks where the Periodic Table of Elements plays an important role. We also deal with the relation between chemistry and mathematics, always from a recreational point of view.

A matter of perspective
Richard Nowakowski, Dalhousie University

Perspective is a tool for the artist and for a crafty problem setter/solver. More mathematicians should use it. I'll give three examples of where it should/could have been used to good effect.

Tiling boards
José Carlos Santos, University of Oporto

Tiling boards (not necessarily rectangular ones) with domino pieces (or, more generally, with polyominoes) raises many challenging problems. We shall see a few of them in this talk.

On the probability equilibrium of the Egyptian game Senet
Joaquim Eurico Nogueira, FCT-UNL
Fátima Rodrigues, FCT-UNL
Luís Trabucho, FCT-UNL

In this work, the notion of Solid Angle is used to compute the probabilities of obtaining the values associated with the throwing of four rods in the Egyptian game of Senet.

It is also shown that, in the three-dimensional case, by taking the limit when the length of the rod over the radius of its cross section goes to infinity, the probabilities obtained depend on the shape and on the equilibrium position of the centre of mass of its cross section.

Euler's formula and the Pascal triangle
Pedro Freitas, ULisboa

Euler's formula can be generalized to triangulations of higher dimensional spheres. Even though the proof of this fact is complicated, one can easily guess the number for each dimension, with some knowledge of the Pascal triangle.

Carl Friedrich Gauss and the formula for the sum of the first n natural numbers
Aleksandra Ravas

It is likely that rare are those who haven't heard the story from Gauss' (1777–1855) youth: one day his primary school teacher asked the pupils to calculate the sum of the first 100 natural numbers. Gauss quickly announced the result was 5050, which is correct. How did he manage to do that? What

was the exact task, to find sum of the first hundred natural numbers, or some other consecutive hundred numbers, as some sources cite? Did Gauss find the formula by himself during the class, or perhaps he knew it all along? The aim of the author is to try to answer to those questions.

How a magician hacks the brain
Adrien Lochon

An thought experiment about the "software" that the human brains runs from the point of view of a computer programmer, followed by some examples of how magicians "hack" this software to make it process reality in a magical way.

Pythagoras, Archimedes, and the napkin ring
Colin Wright, Solipsys Ltd

A puzzle that is now quite well-known involves drilling a hole through a sphere, and then asking a sensible question. In its usual form, the question is posed in such a way as to imply a result that is then assumed, but rarely proved.

In this talk we'll look at that assumption, see why it's true, and then try to find connections to other problems.

Magnums: Counting Sets with Surreals
Peter Lynch, University College Dublin

Cardinality is a *blunt instrument*: the natural numbers, rationals and algebraic numbers all have the same cardinality. So, $\aleph_0$ fails to discriminate between them. Our objective is to define a number $m(A)$ for subsets A of the natural numbers that corresponds to our intuition about the size or magnitude of A.

The class of surreal numbers, denoted by **No** or $\mathbb{S}$, discovered by J. H. Conway around 1970, is the largest possible ordered field, with all the basic arithmetical operations, and sensible arithmetic can be carried out over $\mathbb{S}$. Moreover, quantities like $\omega/2$ and $\sqrt{\omega}$ are meaningful. Surreal numbers have attracted relatively little attention, mostly from recreational mathematicians. Given their appealing properties and their remarkable elegance, this is surprising.

Using the class of surreal numbers, we define the *magnum* for a subset of the power set of $\mathbb{N}$. If A is a finite set, then $m(A)$ is just the cardinality of A. The magnum of a proper subset of a set is strictly less than the magnum of the set itself.

There are difficulties evaluating limits in $\mathbb{S}$. However, the extension of many elementary functions from domain $\mathbb{R}$ to domain $\mathbb{S}$ can be done without difficulty. We know that for the real numbers, $0.999\cdots = 1$. For the surreals, this is not the case; $0.999\ldots = 1 - 10^{-\omega} < 1$. Many more similar examples can be given.

How many odd numbers are there? How many even numbers? From Galileo to Cantor, the suggestion was that there are the same number of odd, even and natural numbers, because all three sets can be mapped in one-one fashion to each other. While this is technically correct, it jars with our intuition, which suggests that there are twice as many natural numbers as odd or even numbers.

We define the density and counting sequence for a set A and the magnum $m(A)$, a surreal number. We then find that $m(\text{Odd}) = m(\text{Even}) = \frac{1}{2}m(\mathbb{N})$. This is consistent with our intuition about the relative sizes of the sets. Several other examples will be presented.

Lewis Carroll's "A Tangled Tale"
Robin Wilson, Open University, UK

Lewis Carroll's "A Tangled Tale" is a series of ten stories that were published in "The Monthly Packet", a Victorian magazine published in England in the 1880s. Each story contains up to three mathematical puzzles hidden within it, which the readers were invited to solve and send in their solutions. In this talk I shall describe several of these ingenious problems and their solutions.

The sweet algorithm
Hugo Almeida

The well known algorithm that finds the maximum common divisor of two natural numbers comes from ancient Greece and bears the name of Euclid. In Portugal, it is mandatory for children of the fifth grade "to use Euclid's algorithm to determine the common divisors of two natural numbers and, in particular, to identify their maximum common divisor." How should we build a bridge between the great Euclid and the 10-year-olds of the 21st century? —Using cakes! Cakes are the natural answer to this enigma. In this presentation we will prove that Euclid had a sweet tooth and that his algorithm can be presented in a playful, accessible way.

A reflection on the ideas of Miguel de Guzman about games and mathematics teaching

Jaime Carvalho e Silva, University of Coimbra

In the talk first given at the "IV Jornadas sobre Aprendizaje y Enseñanza de las Matemáticas, Santa Cruz de Tenerife, 10-14 Septiembre 1984", the late mathematician and former President of ICMI Miguel de Guzman (1936-2004), wrote "El juego bueno, el que no depende de la fuerza o maña físicas, el juego que tiene bien definidas sus reglas y que posee cierta riqueza de movimientos, suele prestarse muy frecuentemente a un tipo de análisis intelectual cuyas características son muy semejantes a las que presenta el desarrollo matemático.". He questions "Dónde termina el juego y dónde comienza la matemática seria?". We will revisit and comment his ideas, connecting with recent proposals of french mathematician and Fields Medalist Cedric Villani.

"Contas de cabeça"–a book of mathematical riddles edited by the Portuguese Football Federation

Hélder Pinto, CIDMA, University of Aveiro
Cristina Silva, Escola Secundária de Pinhal Novo

In the last March we published a book that joined the Portuguese Football Federation and the Portuguese Mathematical Society. In this book, we present several mathematical problems and riddles using football as the common theme. Many of them are classical puzzles from well-known authors in mathematics like Bolt, Dudeney, Loyd, Perelman or Malba Tahan, but now presented in a new and appealing context: football (the passion that enthusiasms almost the entire world). In our presentation, we will show, for instance, how we transformed a Gardner's problem about golden fishes in a problem of goals. The mathematical essence of the problem is the same but we hope that this new approach will be more attractive to younger generations of students. Many times, we hear "that mathematics is everywhere". If that is true, we can always find new contexts to modernize old interesting problems.

Acknowledgments. This work was supported by Portuguese funds through the CIDMA, Center for Research and Development in Mathematics and Applications, and the Portuguese Foundation for Science and Technology ("FCT-Fundação para a Ciência e a Tecnologia"), within project PEst–OE/MAT/UI4106/2014.

An Innovative Card Trick that Combines Classical Concepts

Ricardo Teixeira, University of Houston, Victoria
Jang-Woo Park, University of Houston, Victoria

In this talk we present an innovative card trick that combines two classical mathematical concepts: Si Stebbins deck stack and the Josephus Problem.

A few centuries after the invention of playing cards, magicians began the development of what in some circles is known as a card stacking system, developing what is now known as the Si Stebbins' card system. One of the first known original works was published in 1593 in *Giochi di Carte Bellissimi di Regola e di Memoria*, an Italian book of mathematical card magic, in which the author describes 25 card tricks; many of them based in mathematics. However, it was not until 1630 that some of the material first appeared in English.

In 1896, Si Stebbins, an American vaudeville and circus performer, popularized the system in a book he published under the pseudonym of William Vino, entitled, *William Vino's Card Tricks*. Five years later (1901), Howard Thurston, the leading American stage illusionist of the period, published the same system as his own in *Howard Thurston's Card Tricks*. Despite the sequence having already been used for over three hundred years, each of these two men claimed to be the originator of the sequence.

A general "Si Stebbins" sequence can be defined in mathematical terms, with the aid of Group Theory.

The Josephus problem is named after Flavius Josephus, a Jewish historian in the first century. One account of its invention dates back to the During the Siege of Yodafat which was a 47-day siege by Roman force led by General Vespasian and his son Titus in 67 AD, Josephus, then the governor of the city of Yodafat, and his 40 soldiers were trapped in a cave, refusing to surrender to the Romans. Without hope for escape, they chose to commit suicide rather than be captured and become slaves. Josephus did not agree with the idea of committing suicide, and instead persuaded them to take part in a lethal game of chance, where each soldier would kill the person on their left-hand side, until there is only one survivor. Josephus succeeded to persuade his soldiers, and by chance or by the providence of God, he survived. Then he became a Roman citizen and Jewish historian later known as Titus Flavius Josephus.

The Josephus problem is to find the initial position of the survivor among n soldiers in the scenario described above. The Josephus problem is a main idea behind a famous magic trick: The Penn and Teller's Love Ritual, created by the Spanish magician Woody Aragon.

Our trick combines both concepts in a mentalism performance. Mathematical background and formulation will be presented, as well as quick tips and performance hints.

"The book of numbers": Unveiling sequences
Francisco Albuquerque Picado, ULisboa

In 1986, John H. Conway and Richard K. Guy published "The Book of Numbers", a book where some facets of number theory are popularized (elementary number theory, complex numbers, surreal numbers...) while giving basic and visual proofs of well-established results.

In this talk, I'll explore some techniques introduced in order to study some numerical sequences such as the sequence of the prime numbers and the sequence of the polygonal numbers.

From Gregorian chant to Star Wars: symmetries and mathematical structures in music
Carlota Simões, University of Coimbra
Cristina Cordeiro, University of Coimbra

In his text "Mémoire concernant le calcul numérique de Dieu par des méthodes simples & fausses", Boris Vian (1920-1959) gathers mathematics, literature, religion, satire and humor. He starts with the word "Dieu" (God) and its similarity with the words "Deux" (two) and "Dieux" (Gods). Through numerical calculations - sometimes correct, sometimes wrong, sometimes absurd, but always creative - and of a very own numerology that does not allow translation, the author arrives numerically to the Holy Trinity, raises moral questions about Zero and ends up proposing a nationality to God.

The biggest secrets in the world
Rogério Martins, New University of Lisbon

We will make an excursion through the world of the unknown, and have a glance into the things we know... we don't know. More than a list of open problems we want to discuss how important it is to make the students and general public aware of the frontiers of science.

Are the Chinese Rings Chinese?
Andreas Hinz, LMU Munich

The puzzle nowadays known as the *Chinese Rings* has a long history. There is a description of the solution by Luca Pacioli from the early sixteenth century, and the sequence of minimal numbers of moves (with respect to the number of rings) was presented in 1769 independently in Germany (Georg Christoph Lichtenberg) and Japan (Arima Yoriyuki). A mathematical theory by Louis Gros (1872) triggered Édouard Lucas's invention of the *Tower of Hanoi*. However, the variety of names given to the linked rings throughout Europe and the Far East, like, e.g., Nürnberger Tand, Baguenaudier, Meleda, Jiulianhuan &c., leave the question about its origin open.

NMATH
Rodrigo Girão Serrão, MMA–IST

NMATH is a group of mathematical students that work together because their love and commitment for mathematics drives them to do something more, other than studying. I will show you some of the things we did, do, and intend to start doing!

Defending with one or three dice: What are the RISKs?
Flavia Sancier-Barbosa, Colorado College

The board game RISK has its origin in 1957 (by Albert Lamorisse, France), but its most well-known version was released in 1959 by the Parker Brothers, USA. The game is based on battles between adjacent territories, where the outcome of each battle is determined by the roll of dice. In its 1957 version, the defense used up to three dice in a battle, while in the Parker Brothers' version the defense used only up to two dice (in both versions the attack used three dice). The game was subsequently adopted by several countries, most of which used the Parker Brothers' version, but some have adopted variants that use three dice in the defense, e.g. Italy (RisiKo!), Brazil (War), and Argentina (T.E.G.). In this talk, we first give a brief overview of the mathematical literature related to the game, which is mostly about probabilities of conquering territories when defending with at most two dice. We then extend some of these results to the case where the defense uses three dice and calculate conquering probabilities under different battle strategies (stopping times) for both versions of the game. The probabilities give insight about best tactics and which game is more dependent on chance alone.

The magical mystery Thue-Morse
Jorge Buescu, ULisboa

Rollup for the most magically mysterious sequence of all. Are you interested in or unending chess matches or in sharing a hoard? Or maybe in tennis tie-breakers or penalty shoot-outs? Or perhaps fractals or delicately balancing integer partitions is more your thing? We've got everything you need, satisfaction guaranteed. The magical mystery Thue-Morse is waiting to take you away—take you today!

Topomagic, always the same trick?
Tiago Hirth, Ludus

Two years ago we presented our intention in bringing forth a book on Topological Magic tricks "Topomagic", in it we explained and performed an effect with a closed loop of string, it is equivalent to Borromean Rings. This time we return with Rubber bands and the famous Ring on a Chain as well as other objects and will explore some example effects and their explanation(s).

Popularizing the first seventeenth century books of recreational mathematics: the "Ozanam Project"
Lisa Rougetet, Université de Bretabne Occidentale

The year 2018 coincided with the 300th death anniversary of the French mathematician Jacques Ozanam (1640–1718). The latter is better known for publishing the trigonometric and logarithmic tables during the 1670s, because they were the most precise tables that were available at his time. For living, Ozanam would give private mathematics lessons, first in Lyon and then in Paris. In 1671 he published a Dictionnaire de Mathématiques followed in 1693 by his four-volume work, Cours de Mathématiques. But, more importantly (for us!), Ozanam—who was also a game lover—wrote in 1694 two volumes of Récréations Mathématiques et Physiques, intended for the young persons to sharpen their mind, in the same way that physical exercises would shape their bodies.

The "Ozanam Project"—whose main goal is to rehabilitate Ozanam's recreations in a modern context in order to show that games have a culture and a history—is supported by Plaisir Maths, a French association devoted to the popularization of mathematics through innovative and playful experiences especially in classroom. In this talk, I will present the project in details with the different activities that have been conducted since the start of the project in April 2018, and I will introduce the future possible events.

Cyclic surprises
José Paulo Viana

Using a calculator, it is easy to create recursive sequences and observe the ensuing terms. Sometimes the results are surprising and stimulate our investigative spirit. We will see what we can discover when we unexpectedly come across a cyclic sequence.

History of recreational mathematics
Tereza Bártlová, Charles University

The presentation will be structured as a summary of important mathematical puzzles, games and entertaining problems over time. I will select objects that have a significant role in recreational mathematics and assign them to a data line, or monitor their progress over time. Despite the fact I will endeavor to describe the maximum of mathematical recreation, there is no clear definition of what does belong to recreational mathematics and what does not. This presentation, however, does not attempt to enumerate exactly all the problems of recreational mathematics but to draw attention to the important ones which we still recall for their importance or which influenced the serious mathematics.

Palindrome
Joaquim Eurico Nogueira, FCT-UNL

A palindrome is a word, number, or other sequence of characters which reads the same backward as forward. The purpose of this talk is to focus on some mathematical aspects of this concept, mainly the "196-quest" or the recent result that all numbers may be written as a sum of (at most) three palindromes.

High dimensioned Gosper-like space filling curves
Andrew Shirtz

In this talk I present an extension of the family of Gosper-like space filling curves to higher dimensions. The Gosper curve is unique among other space filling curves as it has a fractal boundary and lies along a triangular lattice. By utilizing an addressing scheme that associates p-adic numbers with the An* lattice, which is analogous to the triangular lattice, I describe a process to produce space filling curves which are Gosper-like in that they have fractal boundaries and lie along the An* lattice for dimensions n where $(n+1)$ and $2^{n+1}-1$ are co-prime.

Mathematical Circus clown tricks
Andreia Hall, University of Aveiro
Paolo Vettori, University of Aveiro
Paulo Almeida, University of Aveiro
Sónia Pais, Instituto Politécnio de Leiria

The Mathematical Circus project was created in 2011 by the LUDUS association with the main goal to promote the interest and motivation for learning mathematics. The Mathematical Circus team performs mathematical magic shows where complementary skills are merged to produce a high intervention capacity within a wide geographical range in Portugal.

Some tricks are performed by a mathematical clown, a unique character in the circus world, who brings together the usual foolish characteristics of a clown with the deep and rigorous mathematical knowledge. In this presentation we will perform and explain some of the tricks of the Mathematical Circus repertoire involving the clown.

Cumulative Subtraction Games
Urban Larsson, Technion, Israel Institute of Technology

We discuss surprising features of a new (but should be old) class of 2-player subtraction games with alternating moves. These games are variations of the classical combinatorial game of nim, but where you instead collect points for the number of pebbles you pick. We prove periodicity results on the outcomes, for an arbitrary reward function, and given a finite subtraction set. Sometimes you must remove a smaller number of pebbles to win, and we make a classification of the situations when such play is optimal. The games were brought to our attention by Fraser Stewart's PhD thesis (2013) and his forthcoming paper "Scoring combinatorial games" in Games of No Chance 5 (Ed. Urban Larsson), and we solve one of his conjectures. Among the remaining open problems, the most intriguing is if already the general class of 2 heap games are algorithmically undecidable. These games exhibit many beautiful patters. This is joint work with Gal Cohensius, Reshef Meir and David Wahlstedt.

Urban sketching with spherical perspective
António Araújo, Universidade Aberta

"Urban sketching" is the practice of drawing urban environments on location, with elementary tools. It is often useful to be able to draw such environments in a very large field of view. Spherical perspective, in its various forms, is the perfect tool for this. I will show some recent geometric results and methods that allow for the drawing of spherical perspectives using very simple tools, and show how these drawings can later be visualized as immersive panoramas using Virtual Reality rendering engines.

Symmetry in the walls of the Palace of Ramalhão
Alexandre Mena e Silva, CSJ

Palace of Ramalhão, in Sintra, has its origins in a small farm that was enlarged into a palace by Luis Garcia de Bivar in 1470. In the 19th century, this palace belonged to the Queen of Portugal, D. Carlota Joaquina. Today is a private catholic school. D. Carlota Joaquina lived in Brazil and that experience inspired some themes of the paintings of that place. We can find elegant friezes painted or engraved by hand in the inside walls of the palace. Nowadays, students can learn geometry, in particular the concept of symmetry, by looking at the history of their own school. In this talk, we will exhibit some paintings, together with some work done by students of the 9th grade.

Who wins the Gambling Tic-Tac-Toe?
Ravi Kant Rai, IEOR, IIT Bombay
K.S. Mallikarjuna Rao, IEOR, IIT Bombay

Combinatorial game theory (CGT) is a branch of mathematics and theoretical computer science that typically studies sequential games with perfect information. We show the optimal play in a game called Gambling tic-tac-toe. We show various results based on the bid in the first and the second round. We show that the game ends in a draw with the best play. We also see the minimum difference between the budget of the players so that the game has a result.

Real Life Problems and a Vast Conspiracy
Adam Atkinson

Some people say they want to see maths applied to real life. So be it. Also: I will reveal the existence of an international multi-decade conspiracy within mathematics.

Bemposta Palace, Military Academy Museums and laboratory
Felipe Manoel Cabral, Instituto Multidisciplinar/UFRRJ
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Nowadays, many brazilians need to resort to the Unified Health System (in portuguese Sistema Único de Saúde–SUS) to be submitted to elective surgeries (non-urgent), resulting in long waiting lists. Aiming to use Mathematical Modeling as a tool for teaching mathematics to make dynamic and natural the learning process, this work aims to determine the optimum amount of surgery procedure by ICU (Intensive Care Unit) whith different duration and complexity, in order to reduce the waiting time for surgery and maximizing the occupation of operating rooms. For this, was taken into account the surgery time for speciality, cleaning and setup time of a operating room. We understand that Mathematical Modeling requires a study of the possibilities and factors that should be considered when constructing a model. In this way, when we apply this tool in mathematics teaching, we allow the student to develop a critical and investigative sense. Therefore, an activity was developed for the last year of high school, which students were to use colored blocks (produced in EVA), that representing surgeries of different speciality (and different procedure times), to "fit" the surgeries into one board. This board represents the programming of three operating rooms to three days. The experiment uses a playful activity allied

to Mathematical Modeling to stimulate the critical instinct of the student, motivating him to understand how mathematics is present in the daily life. At the same time, promoting the discussion about the problem in question and reinforces the contents learned in the classroom as an inequalities, systems of linear equations, and so on. The results show that managing of demand for surgery and procedure time controls the queue size and waiting time for the surgery, intuitively introducing the concept of optimization.

Bemposta Palace, Military Academy Museums and laboratory
Filipe Papança, Military Academy

Architectural Complex of rare beauty, Royal Palace of Bemposta, the headquarters building of the Military Academy, is the owner of a wealth of architectural heritage, including the main Atrium, the Chapel, the Noble Hall, the Library and the Board Room. The museums in the Palace which include the Library Museum, the Museum of Portable Arms and the Museum of the Chapel of Bemposta, are home to a wealth of collections.

The geometry of the coat of arms of D. Catarina, at the main entrance of the palace, is worthy of special mention, the use of the perspective in the seven panels of tile of the Main Atrium (by Jorge Colaço), in order to highlight each of the Weapons of the Army, the symmetry games of the seventeenth-century tiles with floral motifs from the Noble Hall, the symbolism and decorative motifs of the church-vestments of the Chapel Museum, made in the former Rato Factory.

Both the Library and its museum have rare copies of works of mathematics and engineering by classical authors that are worth investigating, studying and analyzing. These works hold a rare aesthetic beauty, which goes far beyond the subject they teach, possessing some wonderful engravings and making learning a wonderful act of fruition of Truth and Beauty.

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