Fault-free tilings of rectangles by dominos

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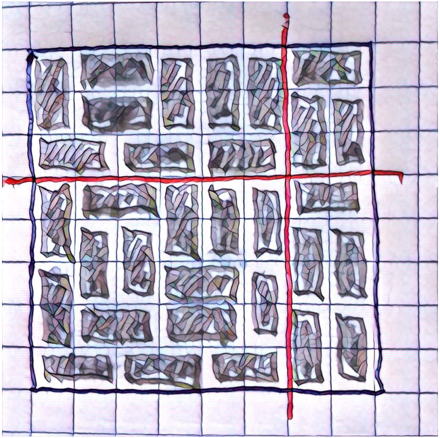
A tiling of an 8x8 chessboard by 32 dominos

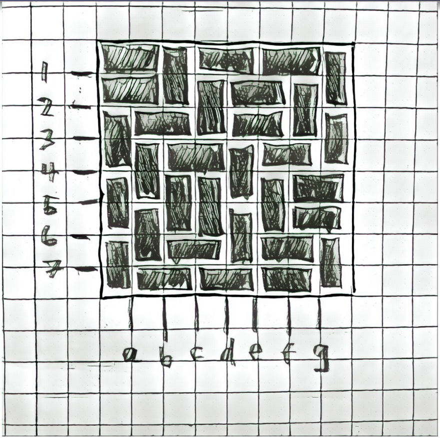
This tiling has two **fault-lines**.

The two fault lines



The two fault-lines in red.

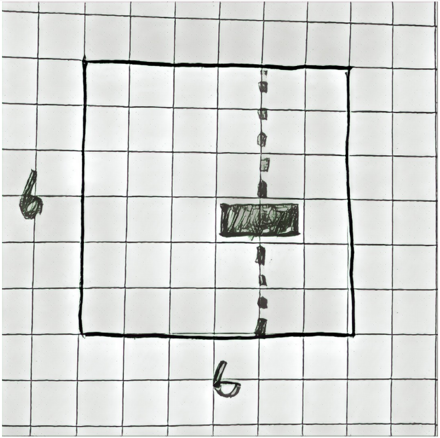
A fault-free tiling of an 8x8 chessboard by dominos



Every fault-line is blocked by a domino.

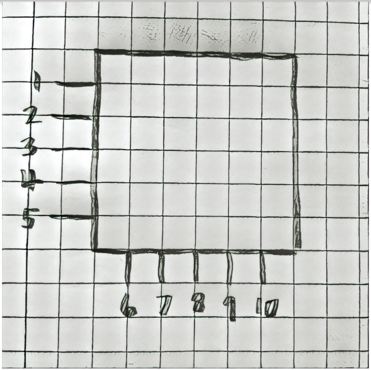
Question: **Is there a fault-free tiling of a 6x6 rectangle?**

A single fault-line of a 6x6 rectangle



Claim: If a 6 x 6 fault-free tiling exists, every line is blocked by at least **two** dominos

Counting dominos in a putative 6x6 fault-free tiling



- So...there are **ten** fault lines to be blocked in a 6x6...
- Each must be blocked by at least **two** dominos...
- So the tiling has at least **twenty** dominos.

General solution (Ron Graham, 1981)

A rectangle with integer sides p and q admits a fault-free tiling by $a \times b$ tiles (where a and b are relatively prime integers) if and only if the following conditions are satisfied:

1. Each of a and b divides one of p and q.
2. Both the Diophantine equations $ax + by = p$ and $ax + by = q$ have distinct solutons in positive integers.
3. If $a = 1$ and $b = 2$, then p and q are not both equal to 6.

For the proof, see Graham's paper in *The Mathematical Gardner*, edited by David A Klarner (1981).