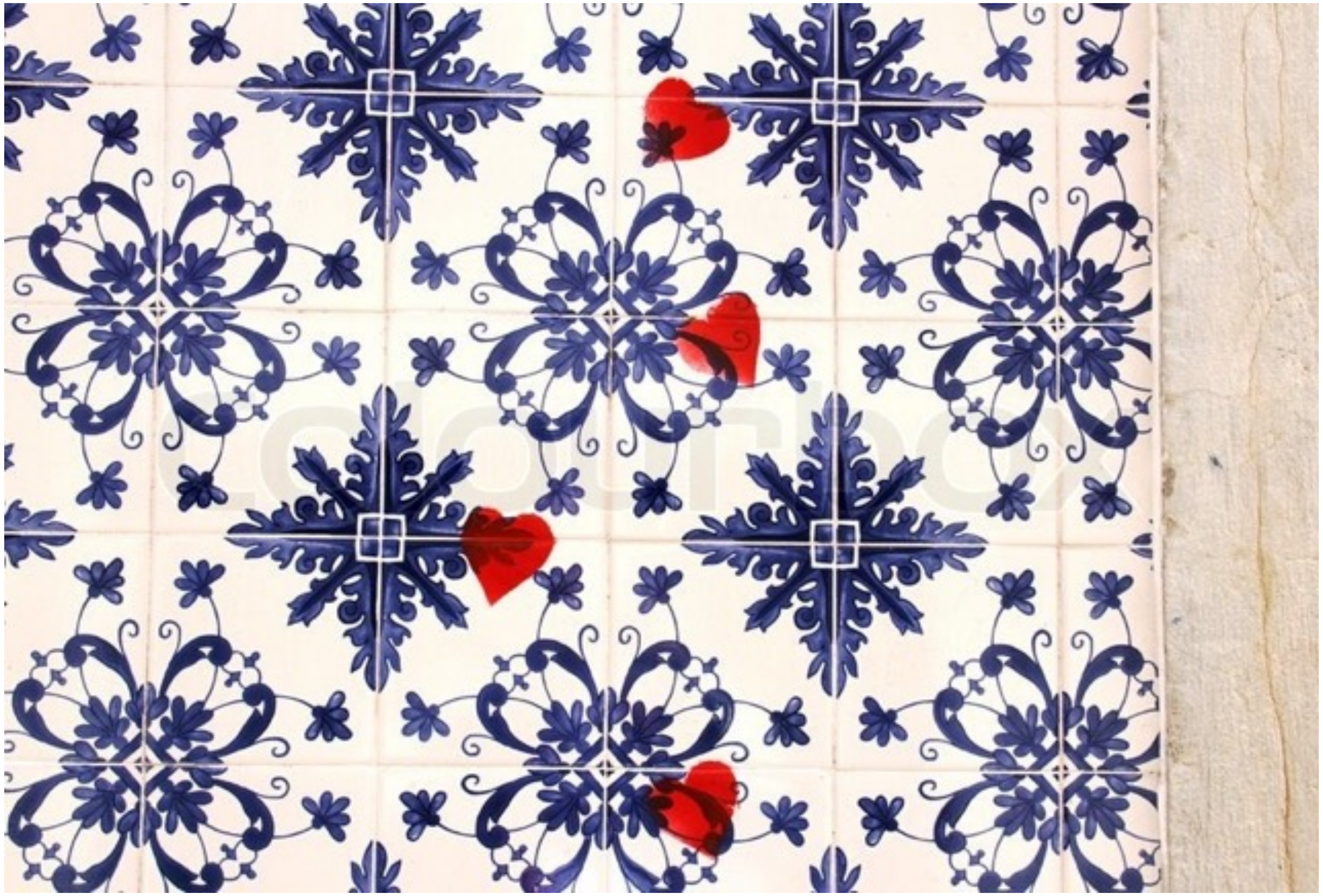
Art in Combinatorial Game Theory

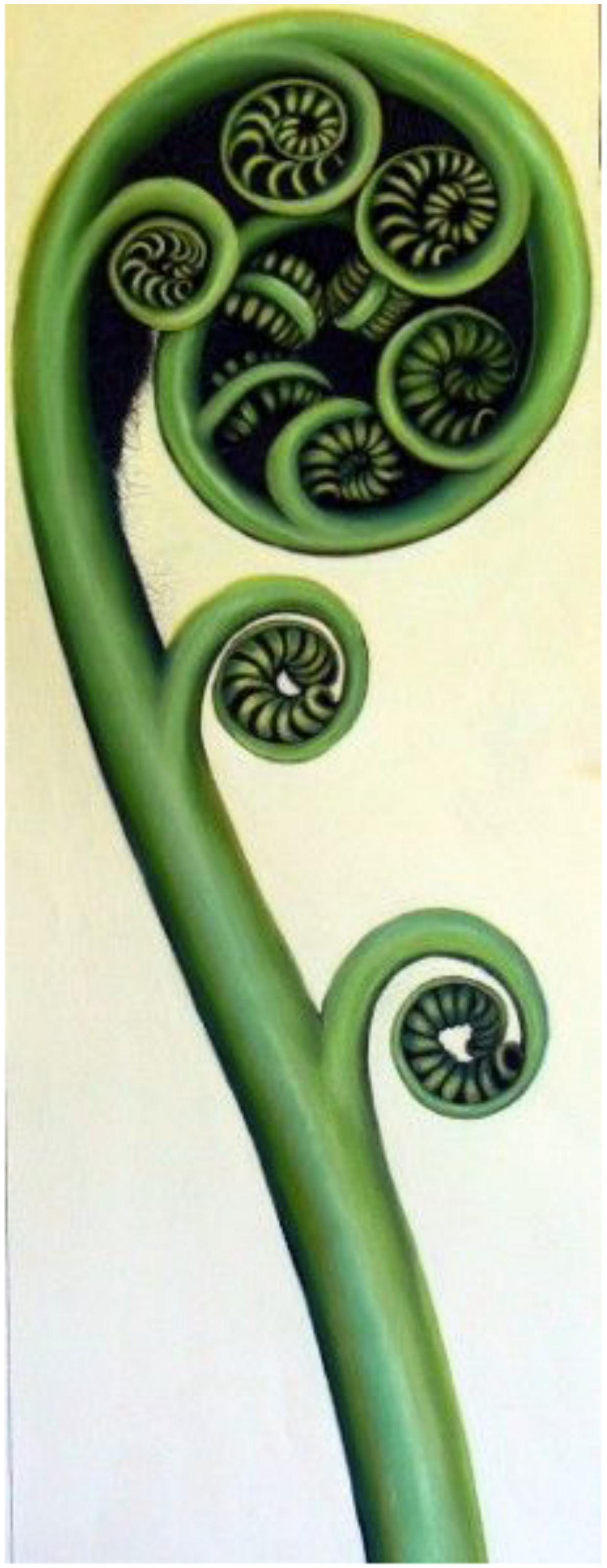
Richard J. Nowakowski

Art in Combinatorial Game Theory

Richard J. Nowakowski




114A



1	1	1
2	$\{1\mid 0\}$	$\{1\mid 0\}$
3	$\frac{1}{2}$	$\frac{1}{2}$
4	$\{1\mid\mid 1\mid 0\}$	1
5	$\{1, \{1\mid 0\}\mid 0\}$	$\{1\mid 0\}$
6	$\frac{3}{4}$	$\frac{3}{4}$
7	$\{1\mid\mid 1\mid 0\mid\mid\mid 0, \{1\mid 0\}\}$	$\{1\mid 0\}$
8	$\{1\mid \frac{1}{2}\}$	$\{1\mid \frac{1}{2}\}$
9	$\{1\mid \{1\mid 0\}, \{1\mid\mid 1\mid 0\}\}$	1
10	$\{1, \{1\mid 0\}\mid 0, \{1, \{1\mid 0\}\mid 0\}\}$	$\{1\mid 0\}$
11	$\frac{5}{8}$	$\frac{5}{8}$
12	$\{1\mid\mid 1\mid 0\mid\mid\mid\mid 1\mid\mid 1\mid 0\mid\mid\mid 0, \{1\mid 0\}\}$	1
13	$\{1, \{1, \{1\mid 0\}\mid 0\}\mid 0\}$	$\{1\mid 0\}$
14	$\frac{7}{8}$	$\frac{7}{8}$
15	$\{\{1\mid\mid 1\mid 0\}, \{1\mid\mid 1\mid 0\mid\mid\mid 0, \{1\mid 0\}\}\mid 0, \{1\mid 0\}\}$	$\{1\mid 0\}$
16	$\{1, \{1\mid \frac{1}{2}\}\mid \frac{1}{2}\}$	$\{1\mid \frac{1}{2}\}$
17	$\{1\mid \{1\mid 0\}, \{1\mid\mid 1\mid 0\}\mid\mid \{1\mid 0\}, \{1\mid\mid 1\mid 0\}\}$	1
18	$\{1, \{1, \{1\mid 0\}\mid 0, \{1, \{1\mid 0\}\mid 0\}\}\mid 0, \{1, \{1\mid 0\}\mid 0\}\}$	$\{1\mid 0\}$
19	$\frac{11}{16}$	$\frac{11}{16}$
20	$\{\{1\mid\mid 1\mid 0\mid\mid\mid\mid 1\mid\mid 1\mid 0\mid\mid\mid 0, \{1\mid 0\}\}, \{1, \{1\mid \frac{1}{2}\}\mid \frac{1}{2}\}\mid 0, \{1\mid\mid 1\mid 0\mid\mid\mid 0, \{1\mid 0\}\}\}$	$\{1\mid 0\}$

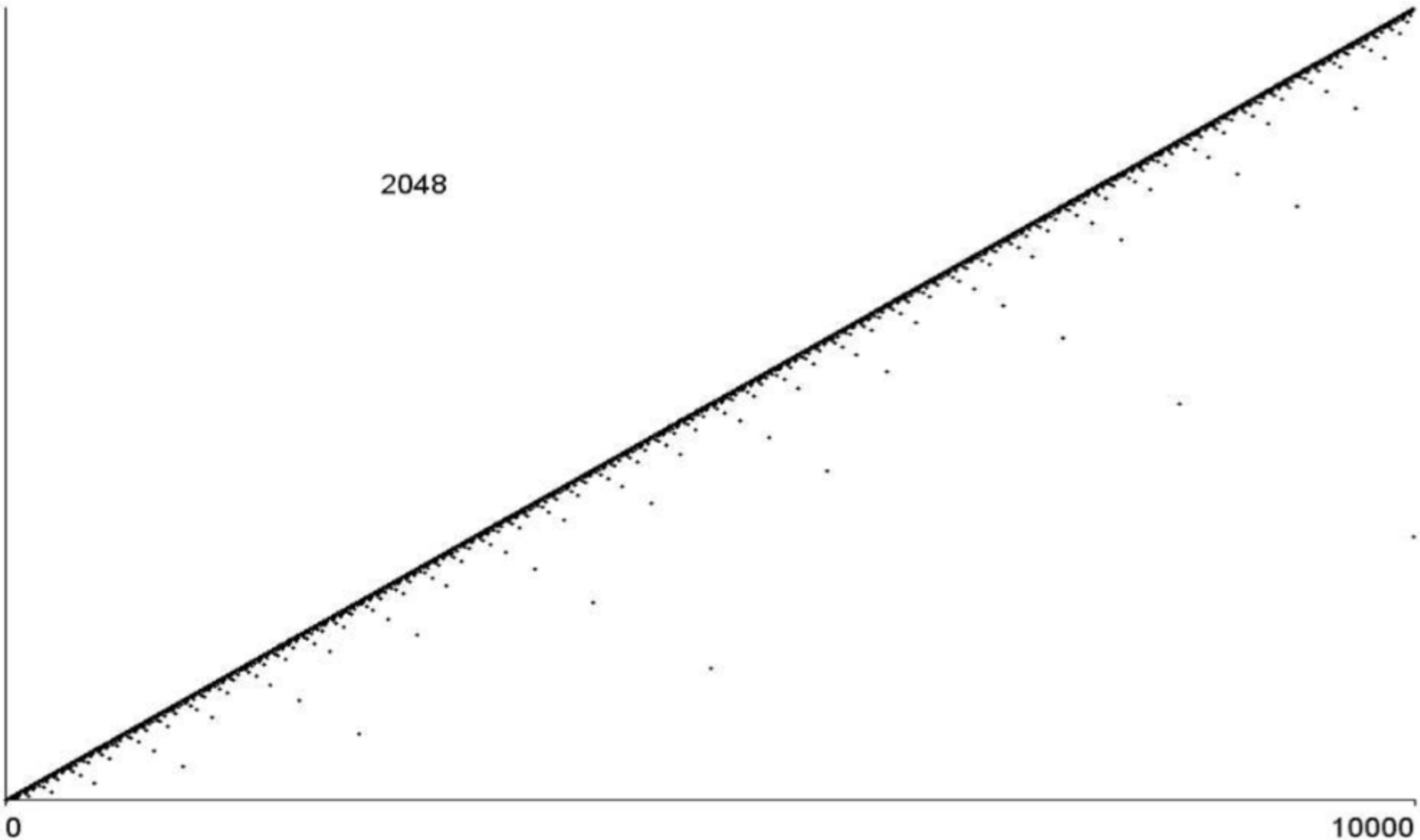
Table 2: The heap values and their reduced canonical forms for some initial heap sizes of GN.

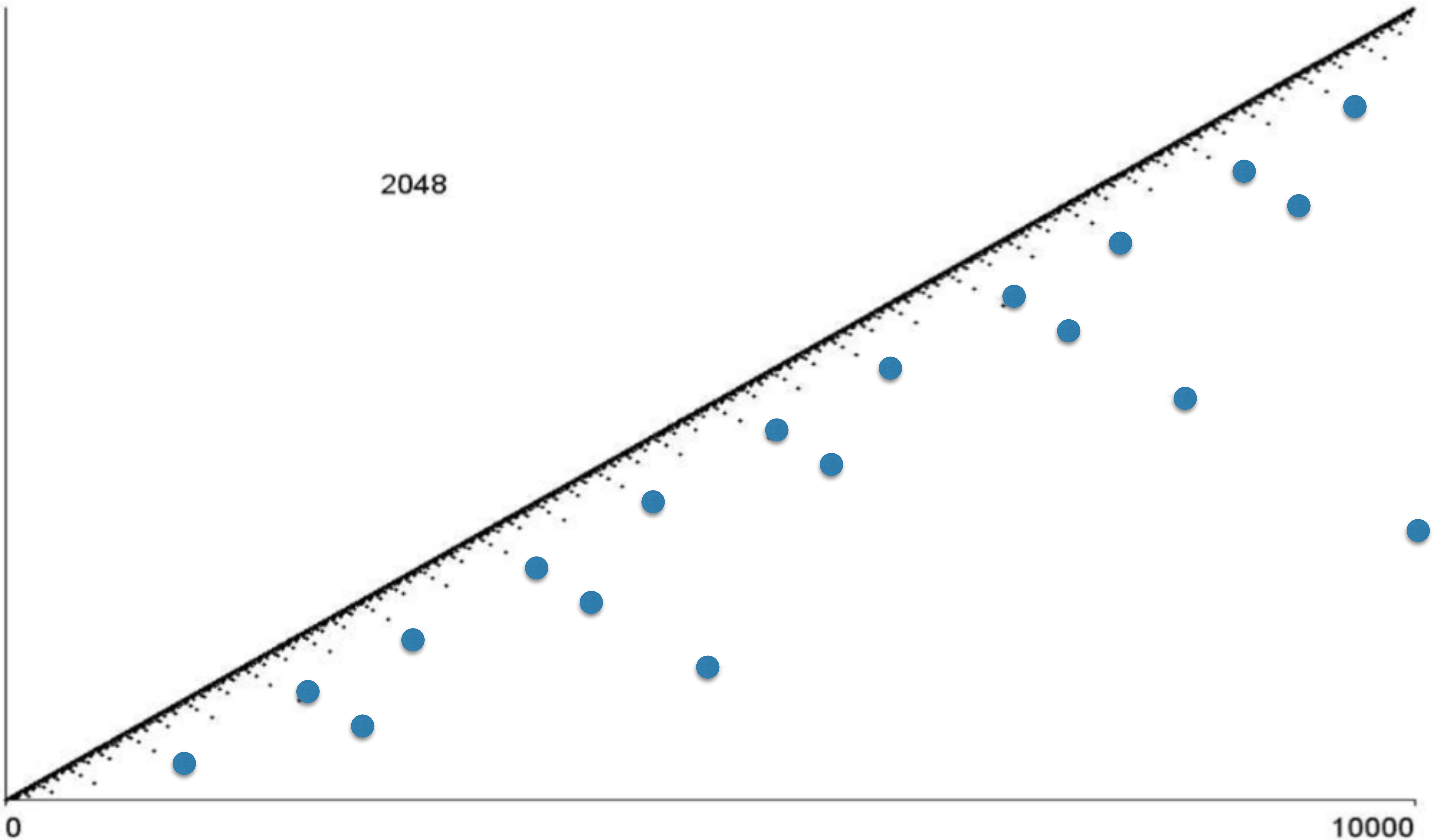
+0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	+11	+12
1												
$\{1\mid 0\}$												
$1/2$	1											

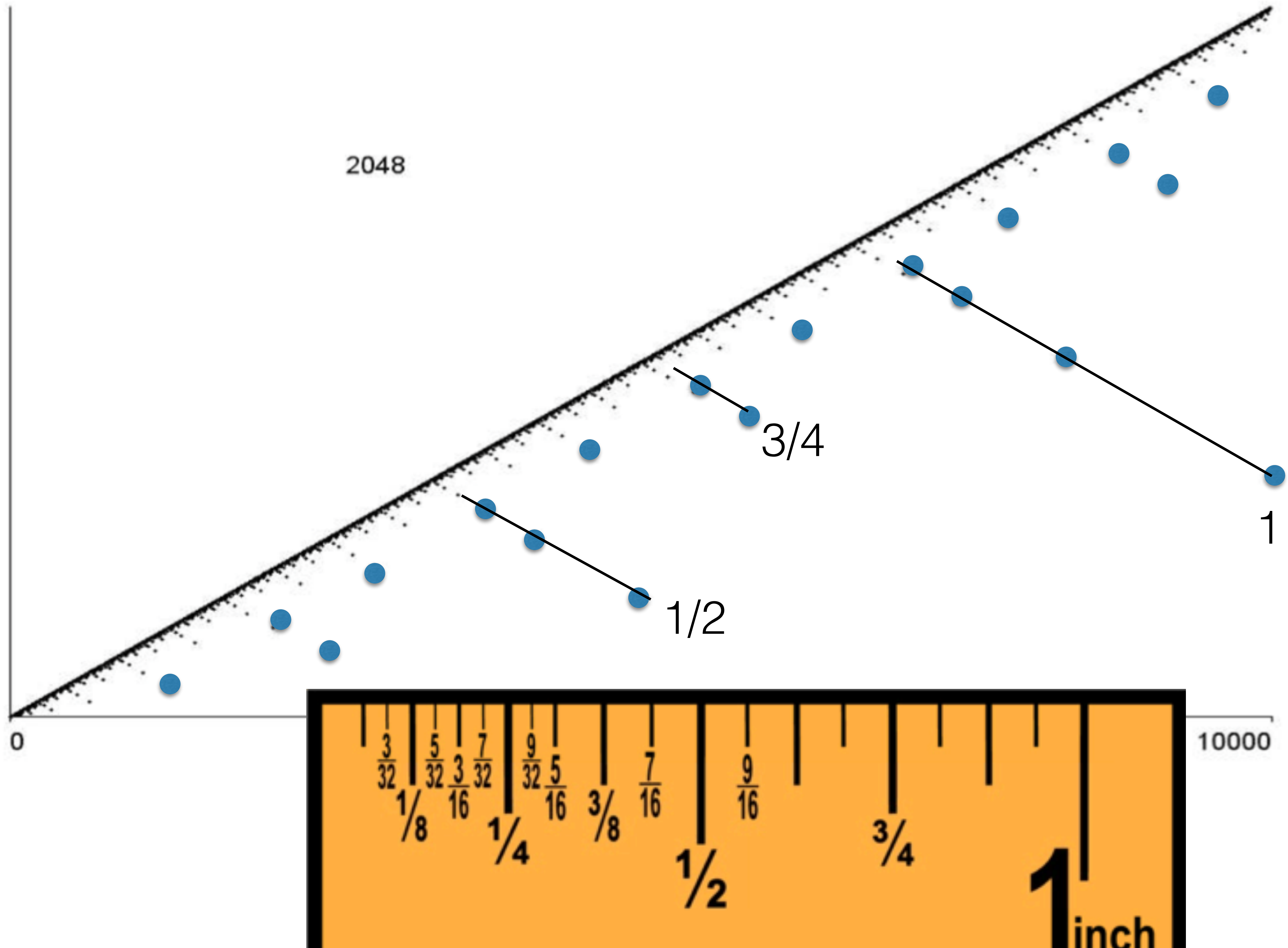
13	$\{1, \{1, \{1\|0\}\|0\}\|0\}$	$\{1\|0\}$
14	$\frac{7}{8}$	$\frac{7}{8}$
15	$\{\{1\|\|1\|0\}, \{1\|\|1\|0\|\|\|0, \{1\|0\}\}\|0, \{1\|0\}\}$	$\{1\|0\}$
16	$\{1, \{1\|\frac{1}{2}\}\|\frac{1}{2}\}$	$\{1\|\frac{1}{2}\}$
17	$\{1\|\{1\|0\}, \{1\|\|1\|0\}\|\|\{1\|0\}, \{1\|\|1\|0\}\}$	1
18	$\{1, \{1, \{1\|0\}\|0, \{1, \{1\|0\}\|0\}\}\|0, \{1, \{1\|0\}\|0\}\}$	$\{1\|0\}$
19	$\frac{11}{16}$	$\frac{11}{16}$
20	$\{\{1\|\|1\|0\|\|\|\|1\|\|1\|0\|\|\|0, \{1\|0\}\}, \{1, \{1\|\frac{1}{2}\}\|\frac{1}{2}\}\|0, \{1\|\|1\|0\|\|\|0, \{1\|0\}\}\}$	$\{1\|0\}$

Table 2: The heap values and their reduced canonical forms for some initial heap sizes of GN.

	+0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	+11	+12
1	1												
2	$\{1\|0\}$												
3	1/2	1											
5	$\{1\|0\}$	3/4	$\{1\|0\}$										
8	$\{1\|1/2\}$	1	$\{1\|0\}$	5/8	1								
13	$\{1\|0\}$	7/8	$\{1\|0\}$	$\{1\|1/2\}$	1	$\{1\|0\}$	11/16	$\{1\|0\}$					
21	$\{1\|1/2\}$	1	$\{1\|0\}$	$\{1\|5/8\}$	1	$\{1\|0\}$	13/16	$\{1\|0\}$	$\{1\|1/2\}$	1	$\{1\|0\}$	21/32	1
34	$\{1\|0\}$	15/16	$\{1\|0\}$	$\{1\|1/2\}$	1	$\{1\|0\}$	23/32	$\{1\|0\}$	$\{1\|1/2\}$	1	$\{1\|0\}$	$\{1\|5/8\}$	1
55	$\{1\|1/2\}$	1	$\{1\|0\}$	$\{1\|5/8\}$	1	$\{1\|0\}$	25/32	$\{1\|0\}$	$\{1\|1/2\}$	1	$\{1\|0\}$	$\{1\|21/32\}$	1


2048
0
10000


2048
0
10000


2048
3/4
1
1/2
0
10000
3/32
5/32
3/16
7/32
9/32
5/16
7/16
9/16
1/8
1/4
3/8
1/2
3/4
1
inch

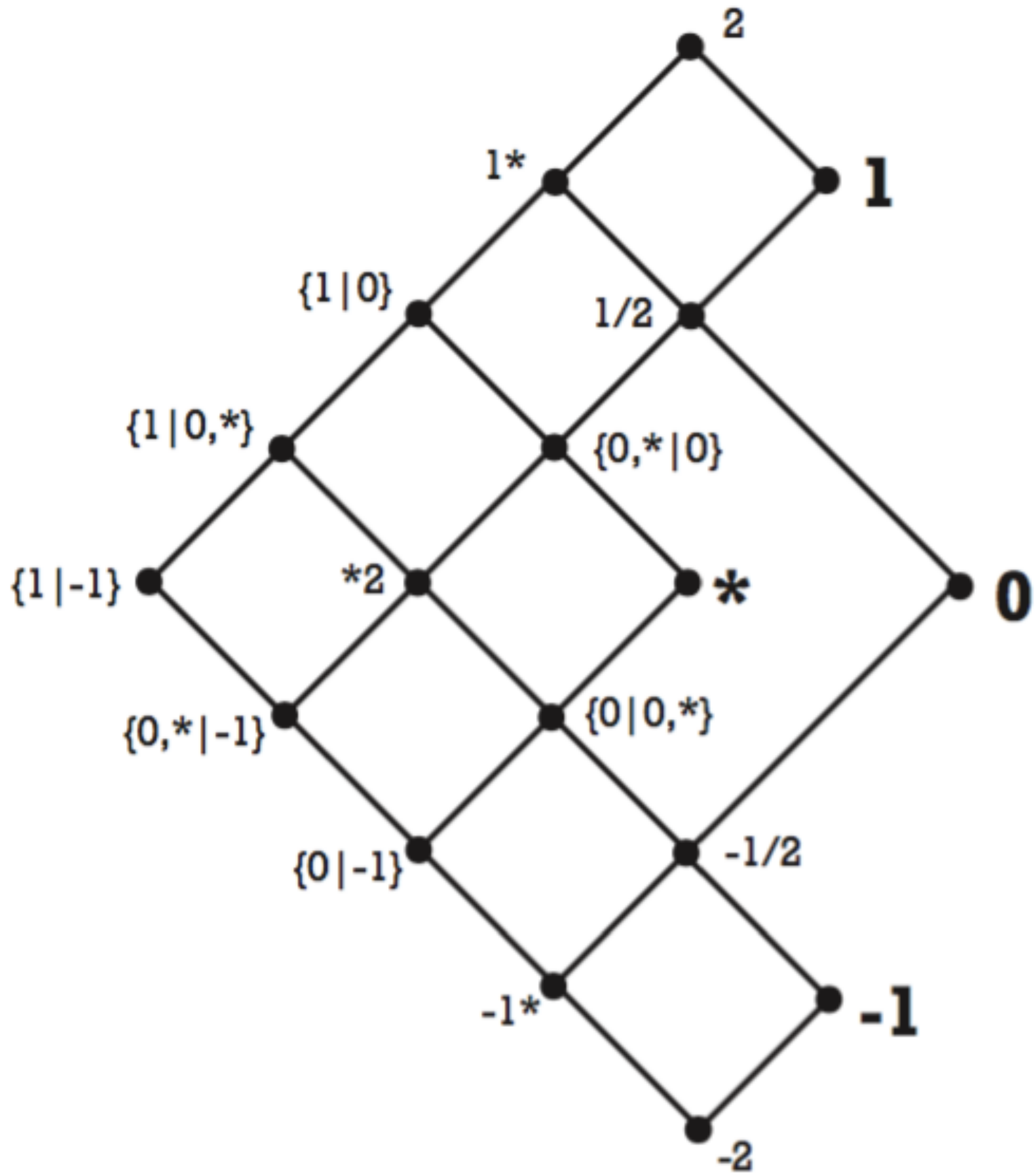
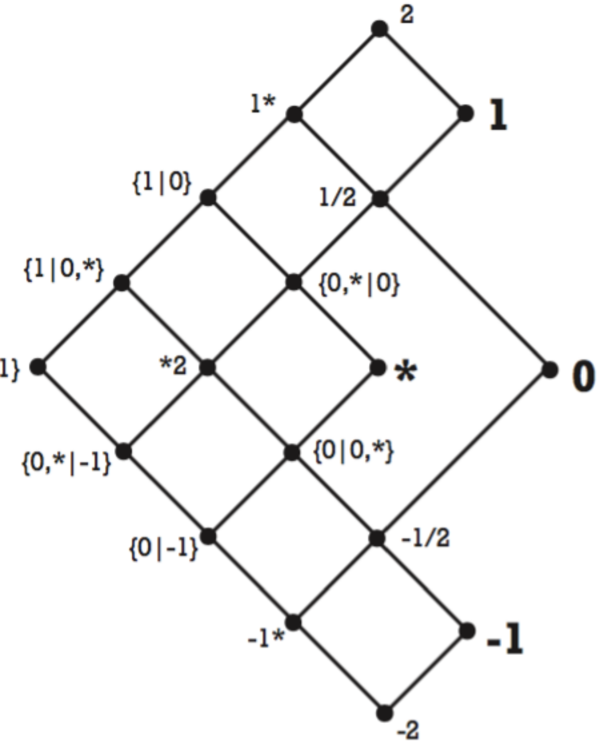
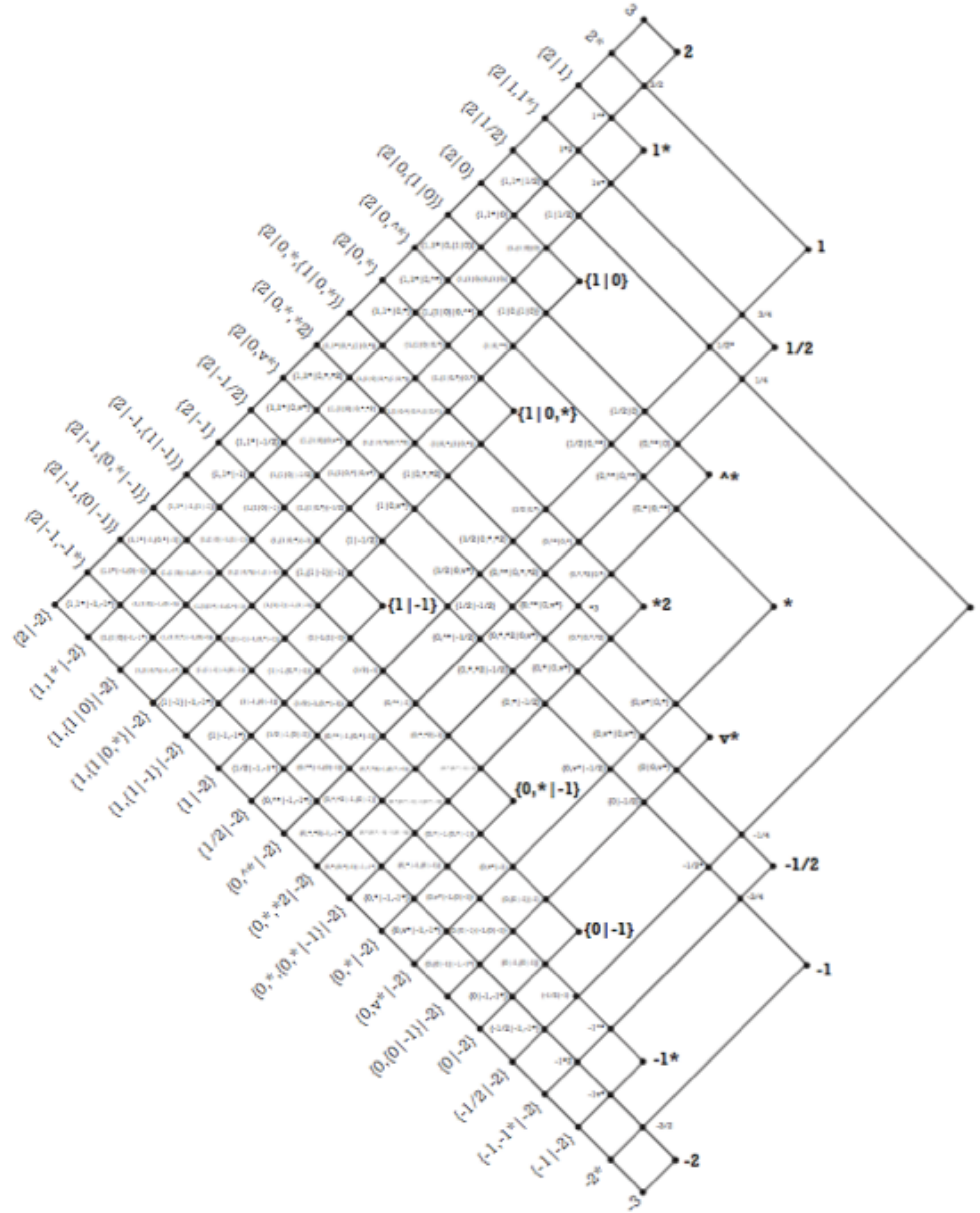


Figure 4.4: The partial-order structure of the 18 option-closed games born by day 2 that make up $\mathbf{OC}_2$.



e partial-order structure of the 18 option-closed games born by day 2
$\mathbf{OC}_2$.



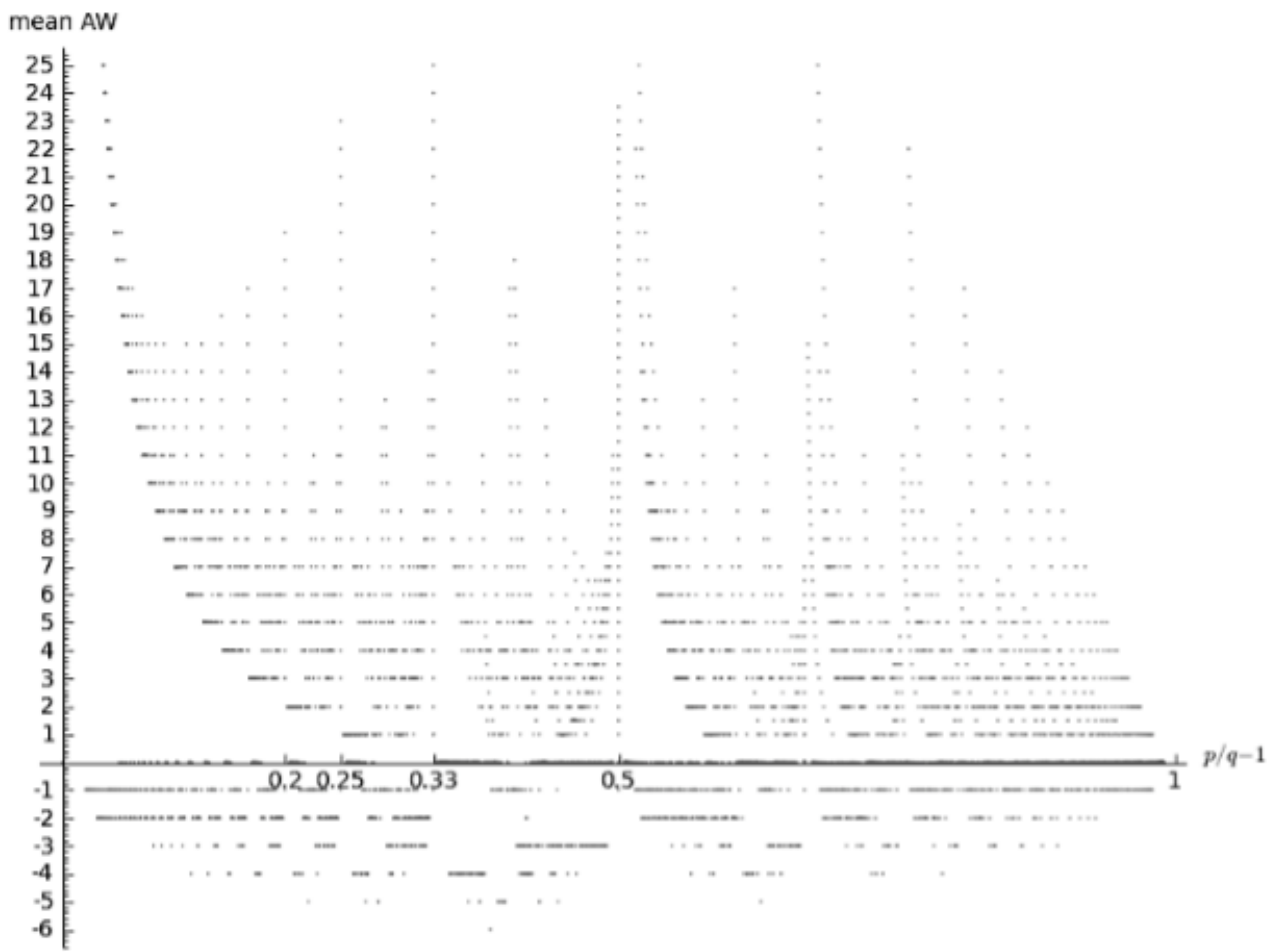


Figure 9.3. Graph showing mean atomic weights of positions against ratio of p to q.

Squashed repeats

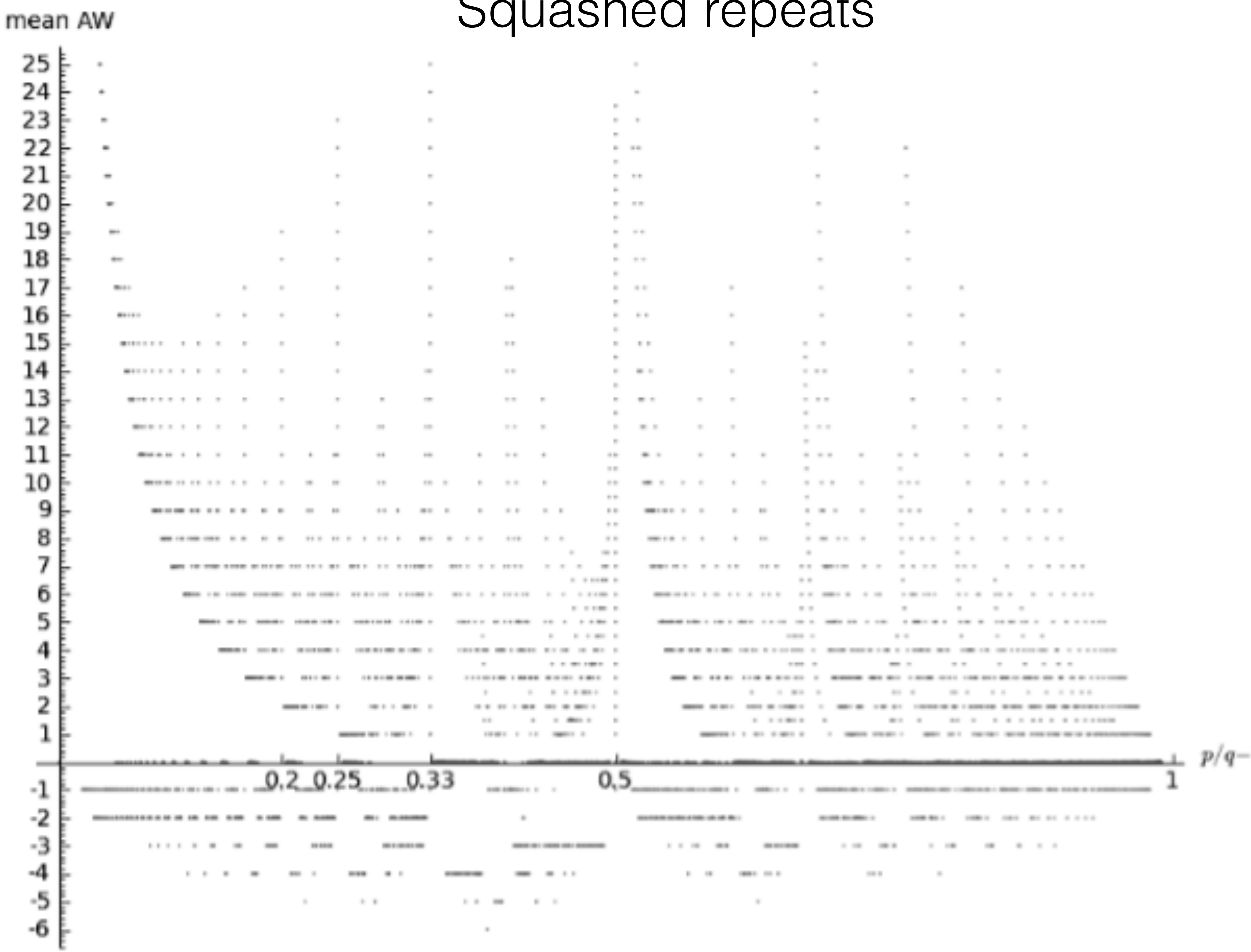
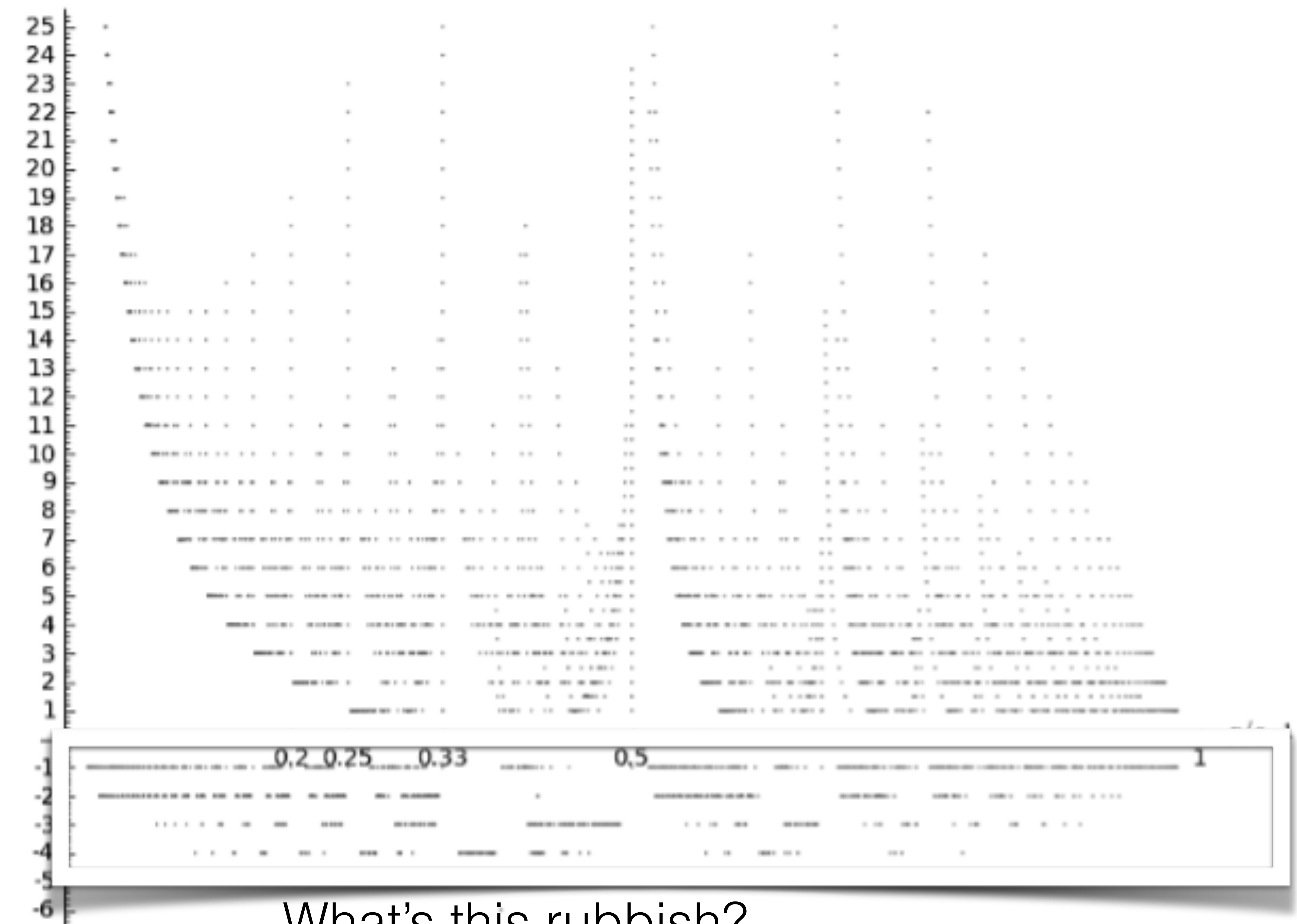


Figure 9.3. Graph showing mean atomic weights of positions against ratio of p to q.

Squashed repeats



What's this rubbish?

Figure 9.3. Graph showing mean atomic weights of positions against ratio of p to q.

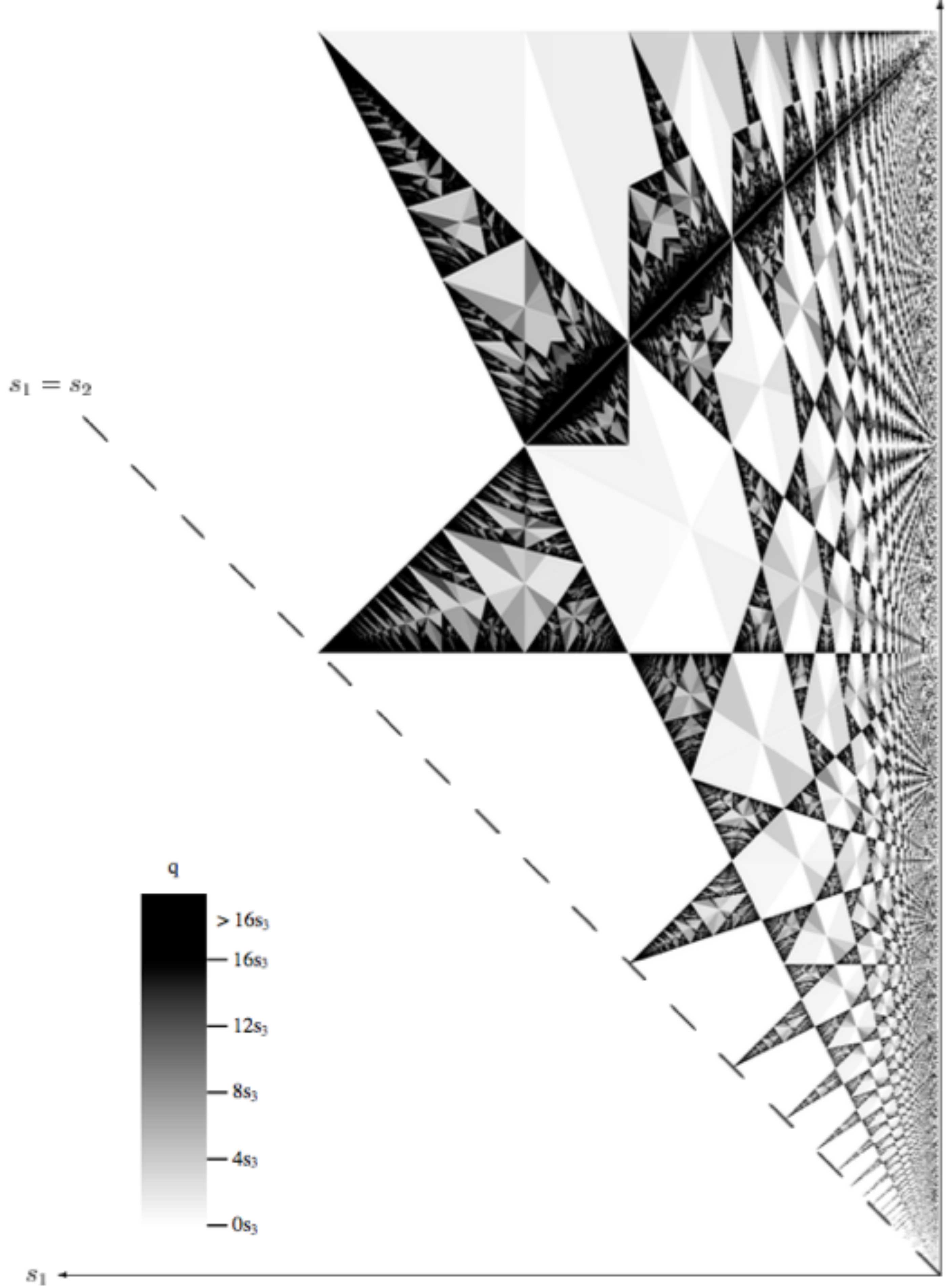
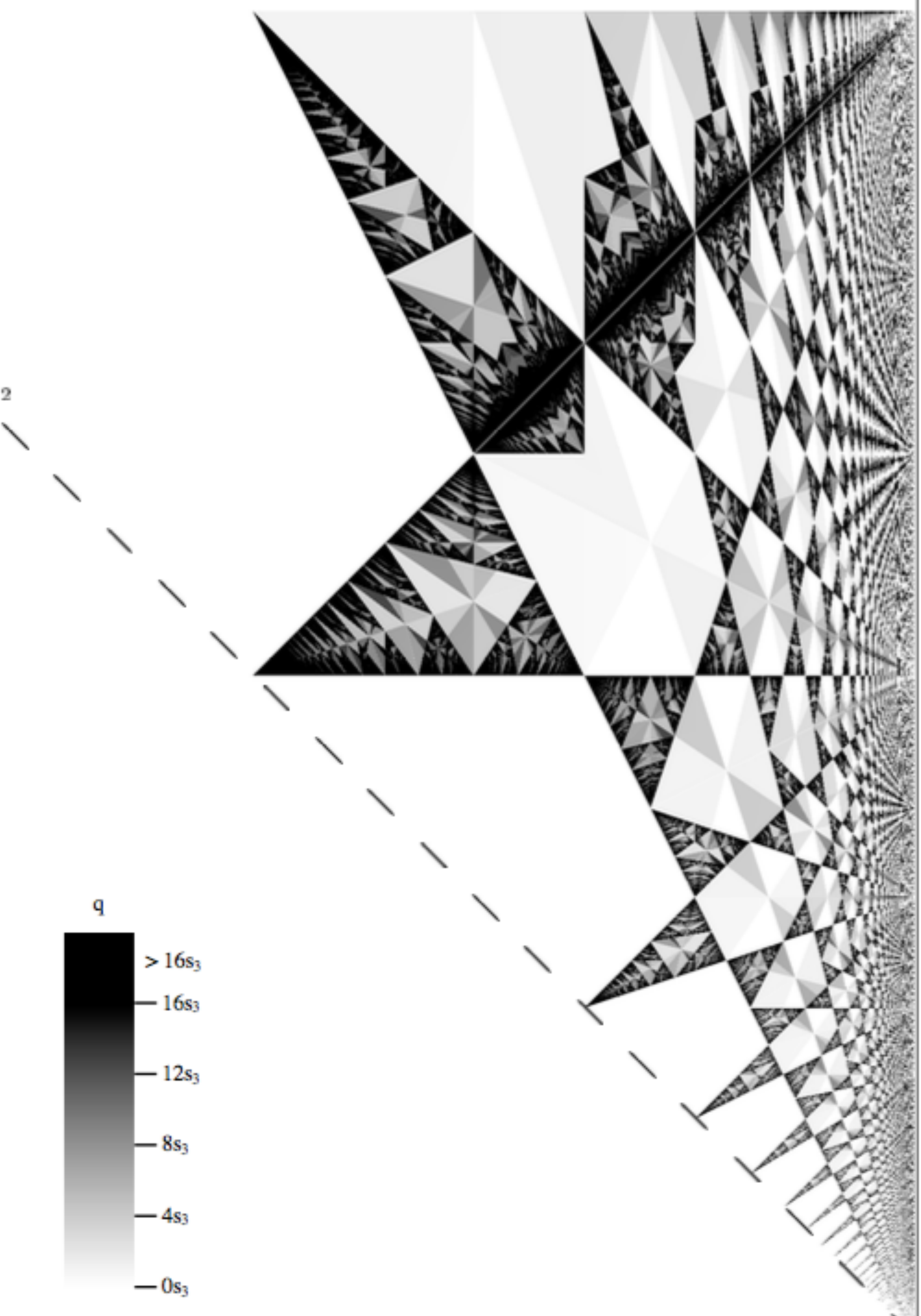


Bild i $s_3 = 1499$ wachsende Vorperiodenlängen sind linear als



2
q
> 16s3
16s3
12s3
8s3
4s3
0s3

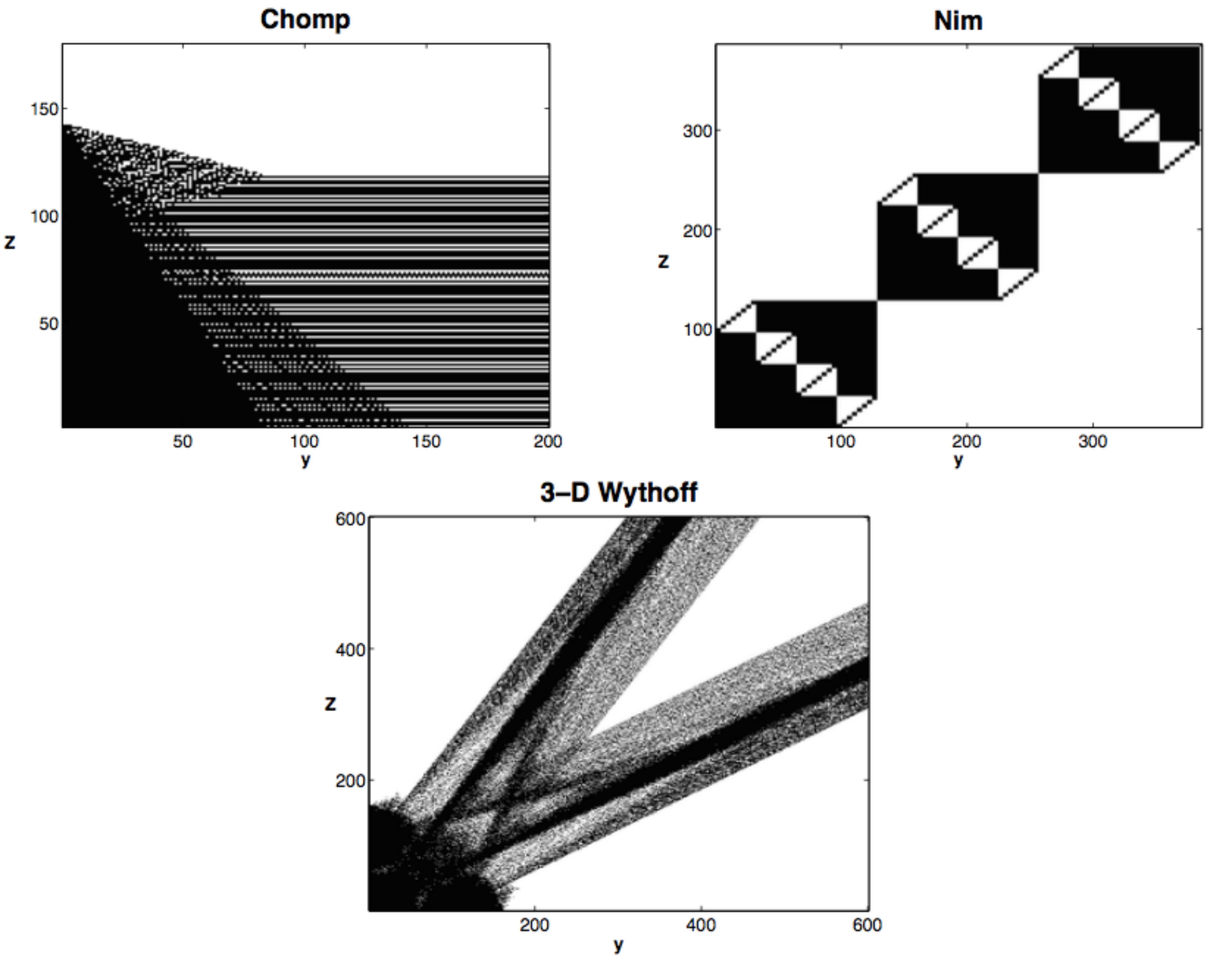


Figure 1. The underlying geometries of combinatorial games. Shown are the IN-sheet structures for Chomp, Nim, and 3-D Wythoff's game.

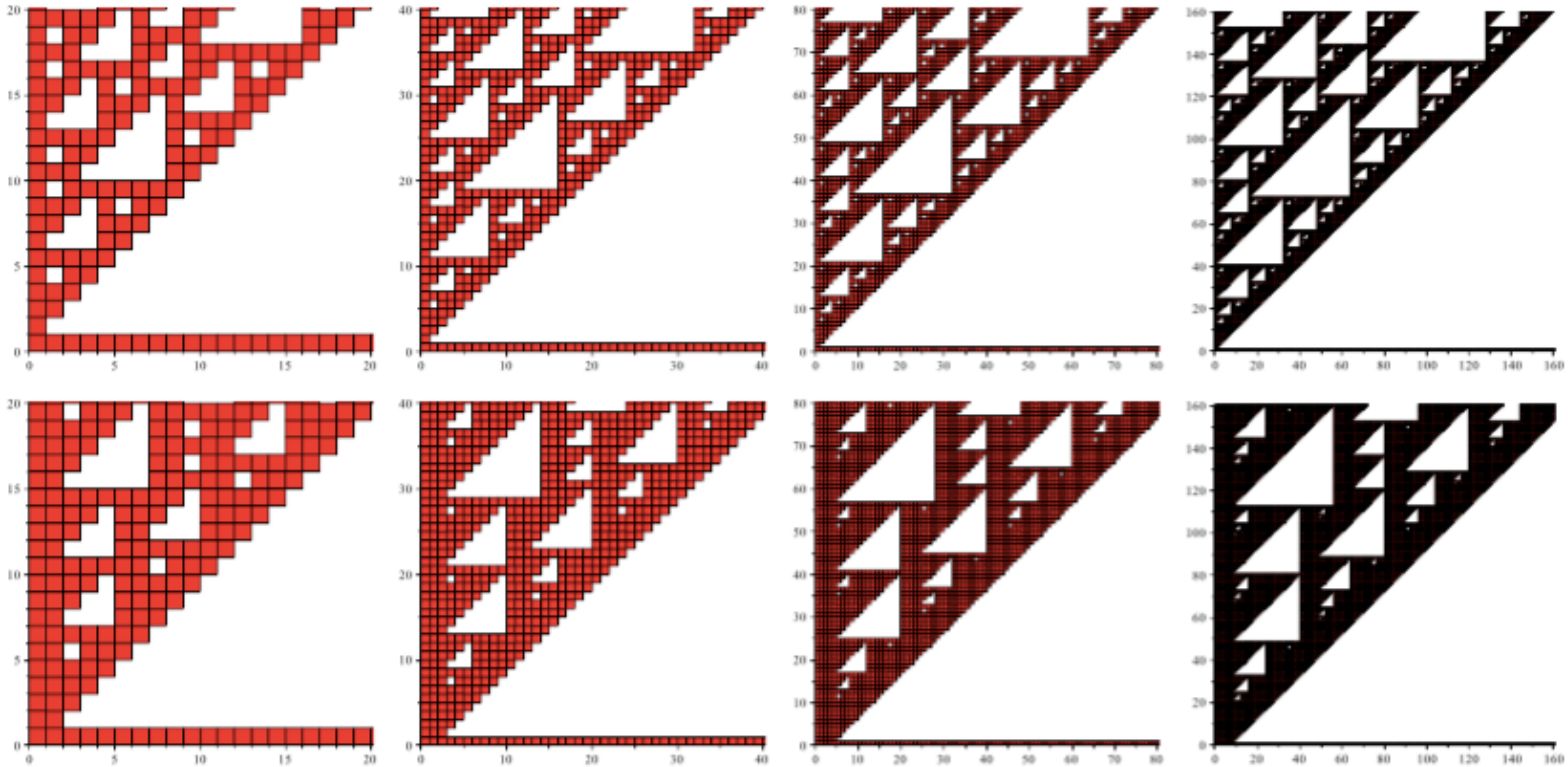
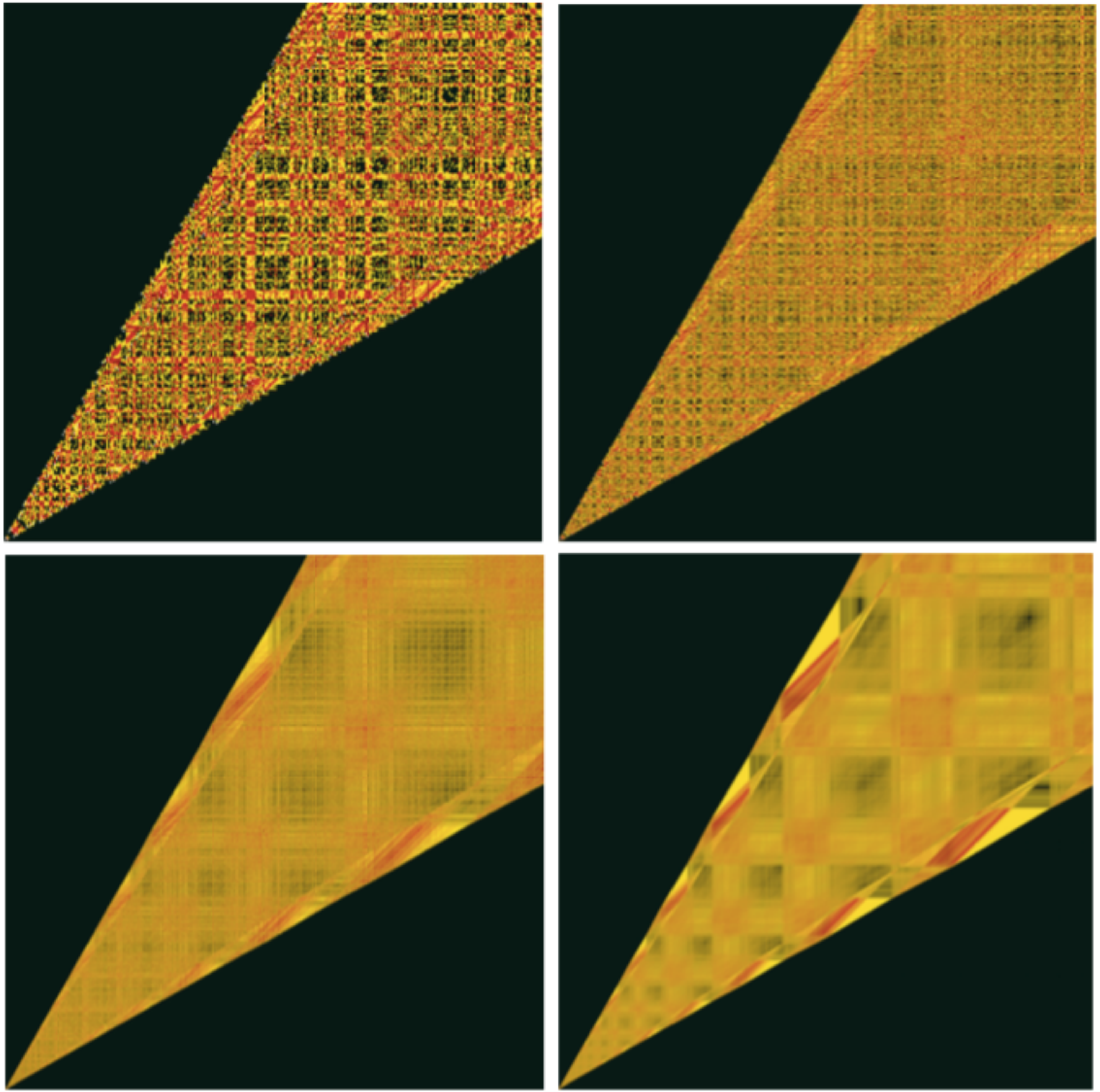
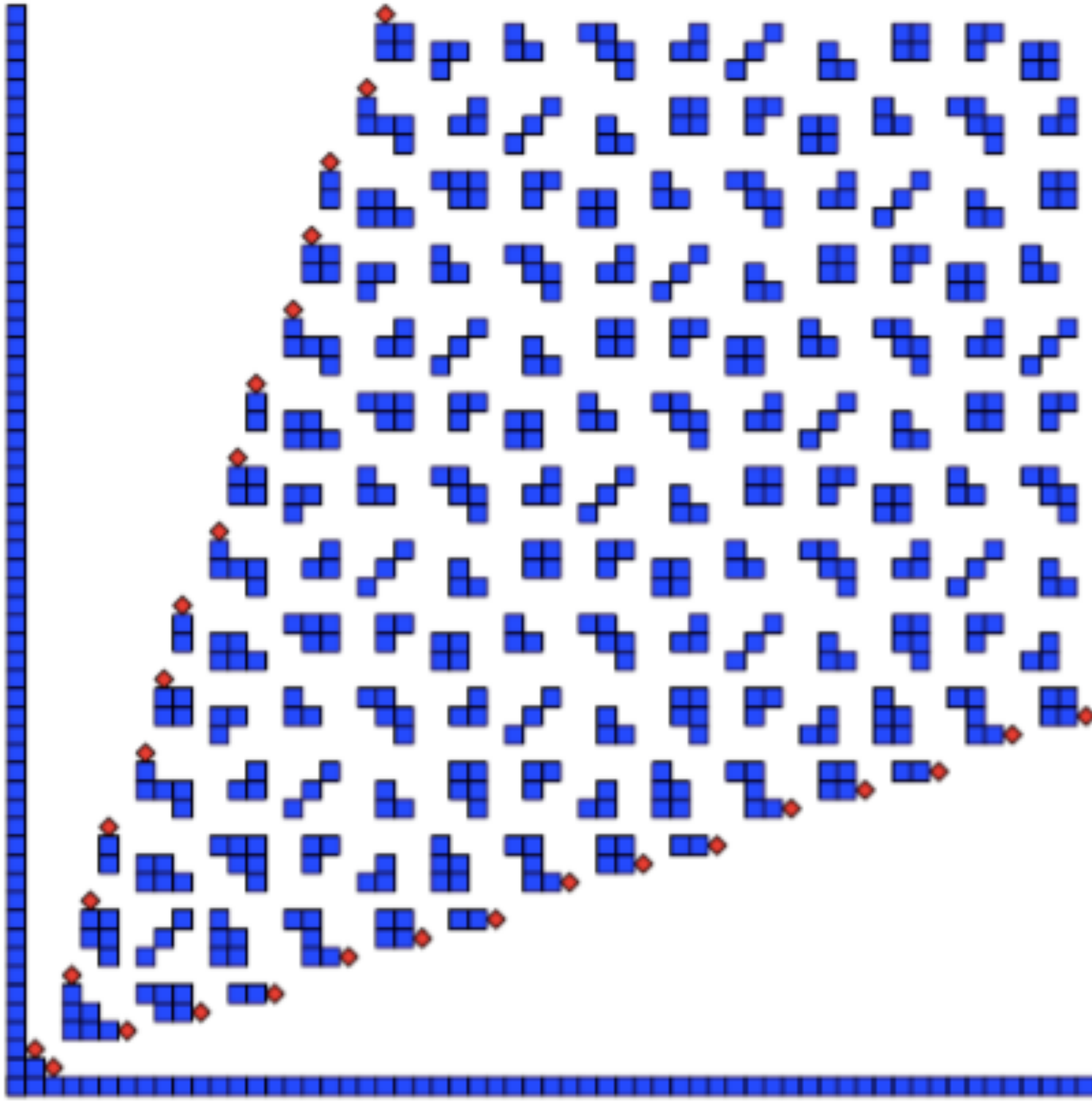
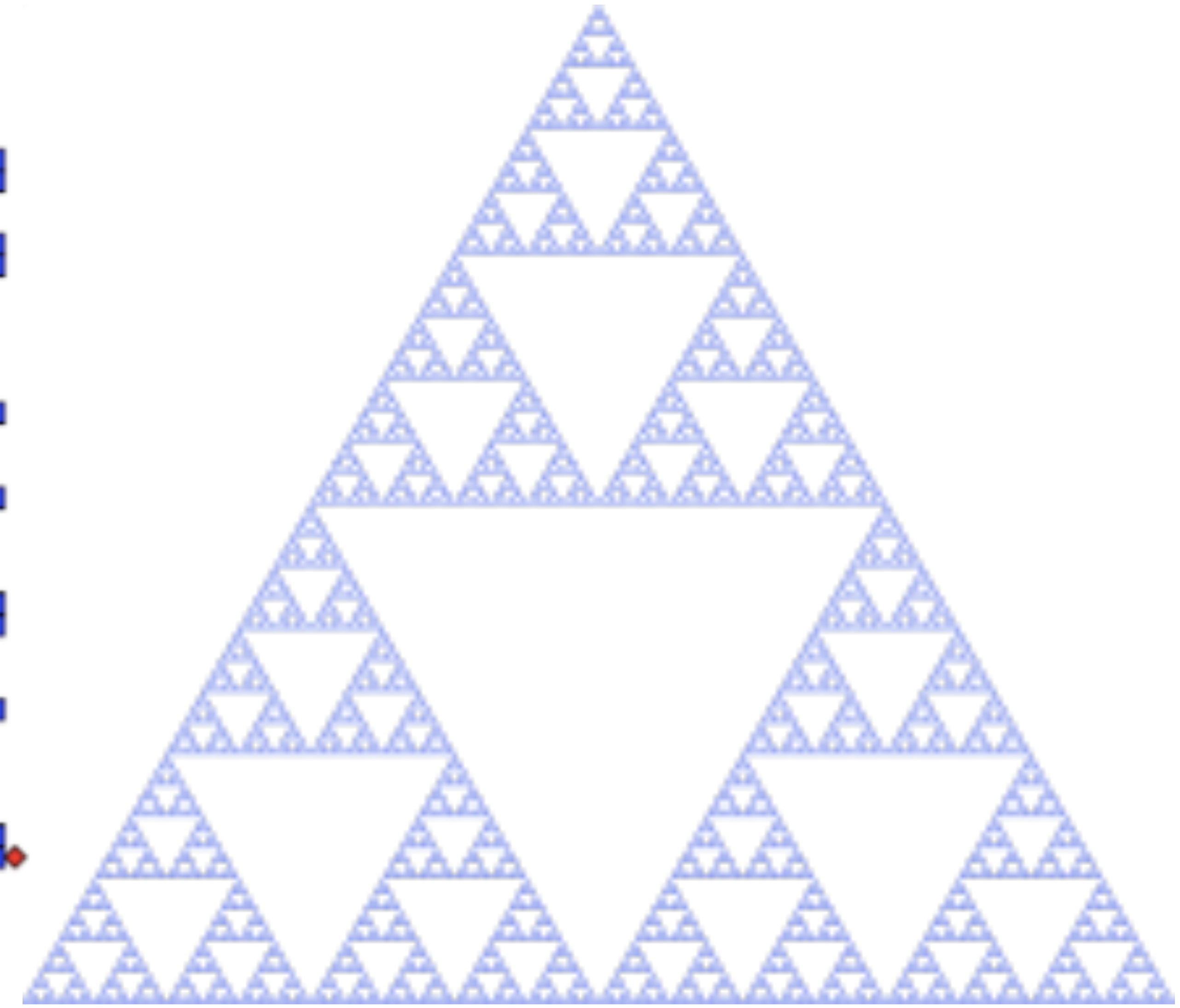


Figure 7: The top figures display $CA_{L,R}$ for $(L,R) = (0,1),(0,2),(0,4),(0,8)$ and those below include $(L,R) = (1,2),(2,4),(4,8),(8,16)$; $\mathrm{CA}_{L,R} = (x,0) = 1$ if and only if $x \geqslant 1$.




these same windowless galleries.

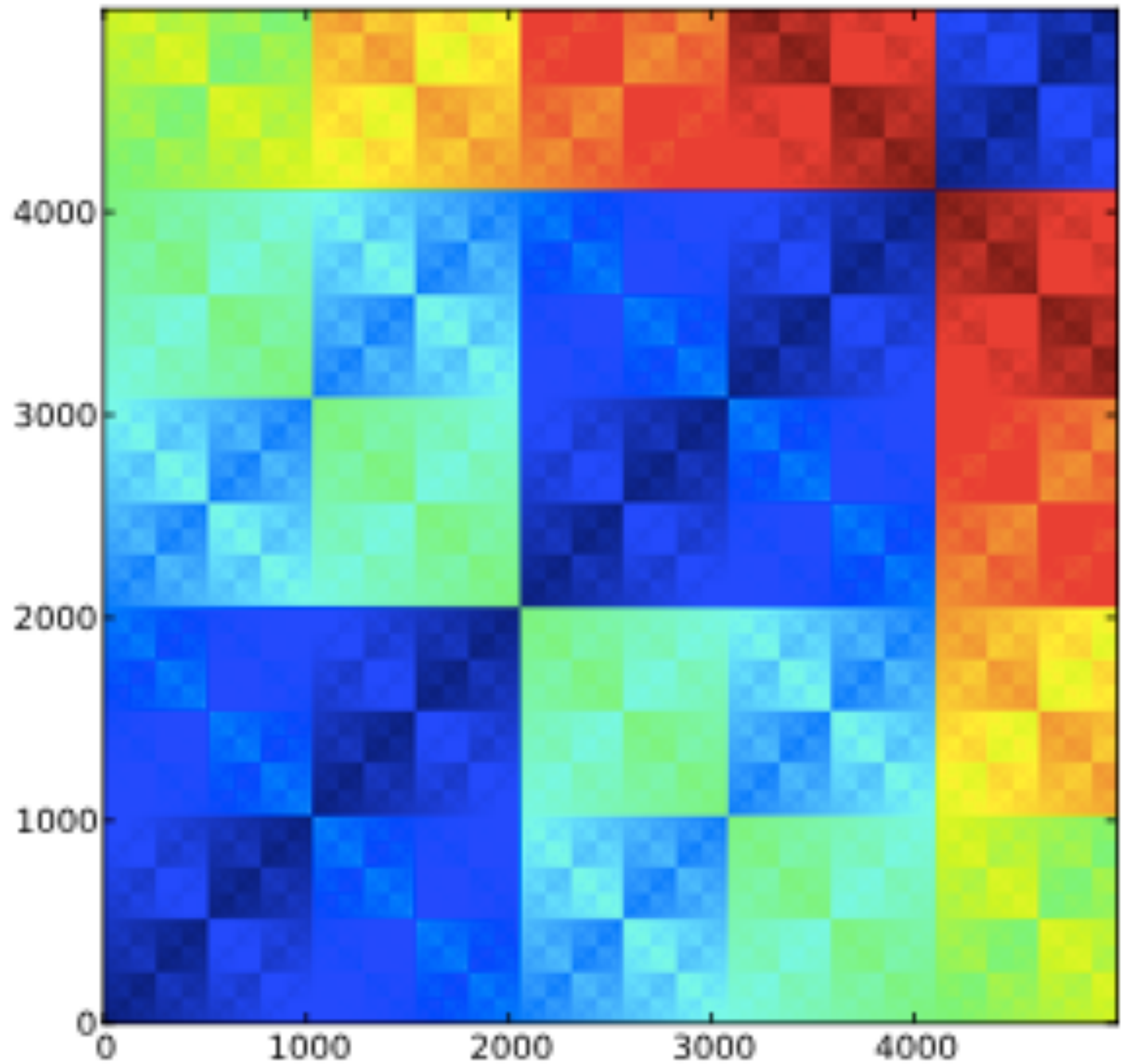




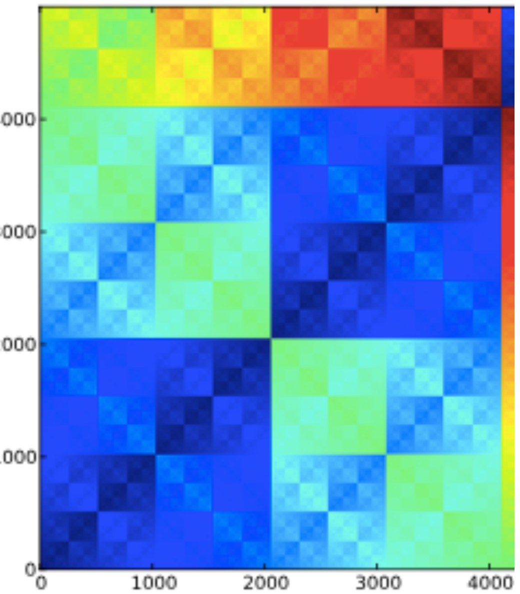
Can lead to this

Or this

OR
EVEN
THIS

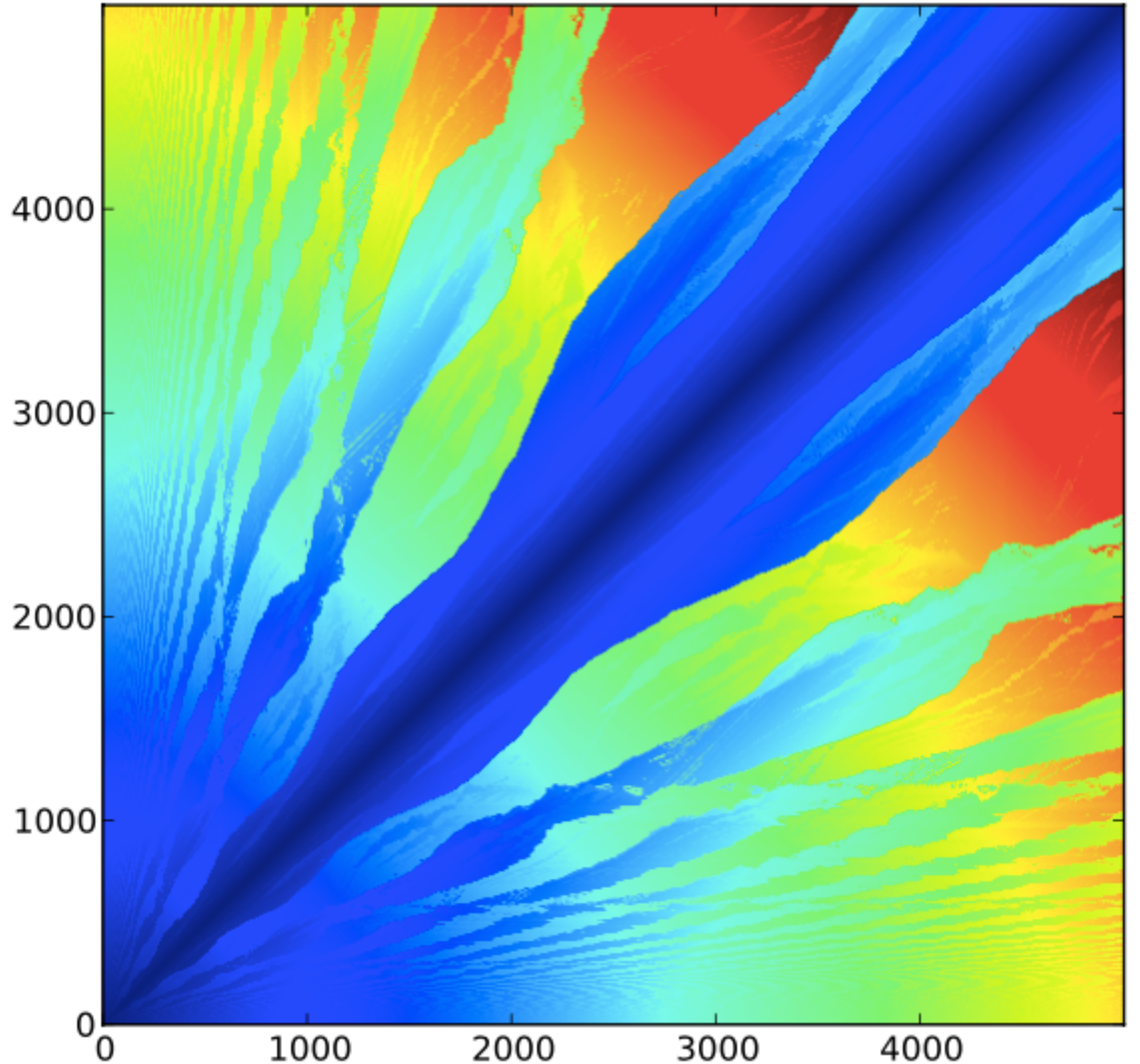


b) The P-positions of Nim of the form $\{x, y, z\}$ with $x, y <$



s of Nim of the form {

OR
EVEN
THIS



(a) The P-positions of Nim-$\{\{1,1,0\}\}$ of the form $\{x, y, z\}$ with